New Type II String Theories With Sixteen Supercharges

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Abstract

We present two new backgrounds of type IIA string theory preserving 16 supercharges. One is a Wilson line for $(-1)^{F_{L_S}}$ and the other is an orbifold by a reflection of four coordinates, along with the action of $(-1)^{F_{L_S}}$, where F_{L_S} is left-moving spacetime fermion number. The Wilson line theory has many new phenomena, including a self-duality of type IIA on a single circle, enhanced gauge symmetry at the self-dual radius, and a T-duality between uncharged and (locally) charged branes. The orbifold theory also presents many novel features, including charged, stable non-BPS D1, D3, and D5-branes pinned to the fixed locus, and an instability of the D0-brane near the fixed locus.

Contents

1 Introduction

String theories with large amounts of supersymmetry have proven useful in probing general properties of string dynamics in relatively controlled models. In this paper we will present two solutions of type IIA string theory[1] which preserve sixteen supercharges. Both theories are exactly solvable at tree level and can be realized as quotients of flat 10-dimensional space.

Both backgrounds involve the operation $(-1)^{F_{LS}}$, which acts as charge conjugation on all RR fields, and as a minus sign on left-moving spin fields $\tilde{\Sigma}$. The first background is a Wilson line for $(-1)^{F_{LS}}$, and the second is an orbifold of four coordinates, where we include $(-1)^{F_{LS}}$ in the orbifold action.

We will study these theories by considering first the closed string theory, then using consistency conditions to derive the allowed sets of boundary conditions in the open string theory. In the case of the Wilson line we begin by using the Green-Schwarz formalism in light-cone gauge to get a quick idea of the spectrum, then transition to the RNS formalism later. Using the GS formalism of course will force us to consider objects which are at least onebranes in the noncompact directions. When we switch to the RNS formalism we will not be forced to adopt that restriction.

Our motivation in studying these backgrounds is that they are simplified models which share some properties of the kinds of nongeometric type II string theories studied in [1]. (For recent related work, see [9].) Namely they are type II theories which have worldsheet descriptions, are worldsheet-chirally asymmetric and have nonzero but reduced supersymmetry relative to ordinary type II models in the same number of dimensions. Indeed, by combining the two orbifolding operations performed separately in this paper, one recovers a limit of the $\hat{c} = 4$ asymmetric orbifold point described in [1] in which three of the four directions decompactify.

Both our theories are quite interesting in their own right, particularly in that all the branes we study are non-BPS but all have at least some range of closed string moduli in which they are stable at tree level. In the orbifold example, some of the non-BPS branes are always stable, and can be related to stable, non-BPS fundamental strings by a duality chain. The backgrounds we describe are fully geometric, and none of the nongeometric ideas of [1] are necessary to understand our results.

[1]Though see [15], [16] for earlier studies of the type IIB version of our second model. The branes of our second model in its type IIA version also appear in the classification [17]. In the language of [17] they are 'truncated' branes with $(r, s) = (1, 0)$. J. Walcher has pointed out to us that both our models can be viewed as orbifolds with discrete torsion. We do not emphasize this language, however, making instead a direct analysis of the consistency conditions for the choice of open and closed string sectors.

2 A Wilson line for $(-1)^{F_{L_S}}$.

Given any exact symmetry g of a particular string theory, it is always possible to construct a background where one direction x^9 is compactified on a circle S^1 of radius R_9, with a Wilson line around that circle which implements the symmetry g on string states. We will consider the case where g is $(-1)^{F_{L_S}}$, the operation which acts with a -1 on all spacetime fermions coming from left-moving spin fields $\tilde{\Sigma}$ in the RNS formalism, or left-moving worldsheet fermions $\tilde{\theta}$ in the GS formalism. g acts with a $+1$ on the corresponding right-moving fields. Consistency then demands that masless p-form potentials are also odd under g, so they will be antiperiodic on the circle x^9.

If we had chosen g instead to be $(-1)^{F_S}$, with F_S the *total* left- and right-moving spacetime fermion number, we would have a conventional Scherk-Schwarz compactification, such as have been studied extensively in string theory [11]. Our Wilson line can be thought of as a kind of chiral Scherk-Schwarz theory.

2.1 Closed strings in the Green-Schwarz formalism

States

In the GS formalism, we take string states to have nonzero p_-, and choose light-cone gauge. The gauge-fixed worldsheet theory has eight transverse bosons X^i with $i \in \{2, \cdots, 9\}$ and eight fermions of each worldsheet chirality. Arbitrary string states with $p_- \neq 0$ correspond to level-matched states of the gauge-fixed theory. Their chirality under the transverse $SO(8)$ is determined by which type II theory we are dealing with. We can always take the left-movers $\tilde{\theta}_a$ to be in the $\mathbf{8}_s$. In type IIB the θ_a are in the $\mathbf{8}_s$ as well, and in type IIA the $\theta_{\dot{a}}$ are in the $\mathbf{8}_a$. In addition to being Weyl with some particular chirality, all our spinors are also Majorana, so they comprise a total of eight real fermionic fields on the left and on the right.

Unlike the worldsheet fermions of the RNS formalism, the $\theta, \tilde{\theta}$ fermions are always periodic on the cylinder.[2] The mass-squared of the string state is just determined by the energy of the state, in units of α':

$$m^2 = 2\ E_{ws}\ r_{ws}/\alpha' = \tfrac{2}{\alpha'}(L_0 + \tilde{L}_0 - 1).$$

Here r_{ws} is the coordinate radius of the worldsheet and $L_0, \tilde{L}_0$ are Virasoro generators of the CFT of the gauge fixed Green-Schwarz worldsheet. With these rules, we calculate

[2]Despite this, vertex operators for spacetime fermions and p-forms do not give rise to cuts in θ. This is not a contradiction, due the lack of symmetry in the light-cone gauge between states on the one hand, which carry $p_- \neq 0$, and operators on the other hand, which carry $p_- = 0$. Vertex operators for states with $p_- \neq 0$ have an involved expression and we do not need to consider them.

the lowest states in the spectrum of the IIA theory. In the untwisted sector, the ground state energy is zero. The zero modes of the θ fermions generate an $\mathbf{8}_s \oplus \mathbf{8}_v$ and the zero modes of the $\tilde{\theta}$ fermions generate an $\mathbf{8}_a \oplus \mathbf{8}_v$. The operator g acts as $g_{\tilde{\theta}} \cdot g_{X^9}$. On states with no oscillators excited, $g_{\tilde{\theta}}$ acts with a $+$ sign on the left-moving $\mathbf{8}_v$ and a $-$ sign on the left-moving $\mathbf{8}_a$ states. g_{X^9} acts only on the zero mode x_9, as $x_9 \to x_9 + 2\pi R_9$, so that it acts as $\exp\{2\pi i p_9 R_9\}$, which gives a $+$ sign on states with $p_9 R_9 \in \mathbb{Z}$ and a $-$ sign on states with $p_9 R_9 \in \mathbb{Z} + \frac{1}{2}$. So the integral KK modes on the circle live in the $\mathbf{8}_v$ sector of the left-moving Hilbert space, and the half-integral modes live in the $\mathbf{8}_a$ of the left-moving Hilbert space.

Let us use the notation n_9 for $p_9 R_9$. The untwisted spectrum with no oscillators excited contains the following states:

- Bosonic states in the $\mathbf{8}_v^{(L)} \otimes \mathbf{8}_v^{(R)}$ of SO(8) with $n_9 \in \mathbb{Z}$. This sector contains the graviton, dilaton, and NS-NS antisymmetric tensor. These states are periodic around the S^1.

- Fermionic states in the $\mathbf{8}_v^{(L)} \otimes \mathbf{8}_s^{(R)}$ of SO(8) with $n_9 \in \mathbb{Z}$. This sector contains the right-moving gravitini and dilatini, all periodic around the S^1.

- Fermionic states in the $\mathbf{8}_a^{(L)} \otimes \mathbf{8}_v^{(R)}$ of SO(8) with $n_9 \in \mathbb{Z} + \frac{1}{2}$. This sector contains the left-moving gravitini and dilatini. These states are antiperiodic around the S^1.

- Bosonic states in the $\mathbf{8}_a^{(L)} \otimes \mathbf{8}_s^{(R)}$ of SO(8) with $n_9 \in \mathbb{Z} + \frac{1}{2}$. This sector contains p-form potentials of rank 1 and 3, and they are antiperiodic around the S^1.

To each of these sectors we add states which are identical except that they contain a winding number $w \equiv \Delta x^9 / 2\pi R_9$ which is even. For $w \in 2\mathbb{Z}$ the boundary conditions on all worldsheet fields are exactly as in the sector of $w = 0$.

Next we consider the twisted sector, which has winding number $w \equiv \Delta x_9 / 2\pi R_9$ equal to 1 mod 2. In the sector with no KK momentum, the winding contributes an energy of $\Delta E_{ws}\ r_{ws} = w^2\ R_9^2/\alpha'$. The $\tilde{\theta}$ fermions are antiperiodic in this sector, so they contribute a ground state energy of $\Delta \tilde{L}_0 = -\frac{8}{16} = -\frac{1}{2}$. There is no corresponding L_0 contribution, so the ground states are not level matched in this sector. The level matched states are obtained in one of two ways: by acting with an odd number of $\tilde{\theta}$ oscillators, since these are half-integrally moded, or by adding a half-integral value of n_9, since fractional momentum contributes to the level mismatch by $w\ n_9$. We can get a level-matched twisted sector by inferring that g has an anomalous phase of -1 in the sectors with odd winding.[3] So the true action of g_{X^9} should be $\exp\{2\pi i n_9 + \pi i w\}$.

[3] In the worldsheet CFT of asymmetric Wilson lines, there is a general prescription for assigning

The oscillator ground states of the twisted sectors contain the following:

- Bosonic states in the $\mathbf{1}^{(L)} \otimes \mathbf{8}_v^{(R)}$ of SO(8) with $n_9 w = -\frac{1}{2}$.
- Fermionic states in the $\mathbf{1}^{(L)} \otimes \mathbf{8}_s^{(R)}$ of SO(8) with $n_9 w = -\frac{1}{2}$.

We can also act with a $\tilde{\theta}_{-\frac{1}{2}}$ oscillator, to level match states with no KK momentum. The $\tilde{\theta}$ oscillators transform in the $\mathbf{8}_s$ of SO(8). Doing this gives the following states:

- Bosonic states in the $\mathbf{8}_s^{(L)} \otimes \mathbf{8}_s^{(R)}$ of SO(8) with $n_9 = 0$. This sector has eight-dimensional masses $m_8^2 = w^2 \ R_9^2/\alpha'^2$.
- Fermionic states in the $\mathbf{8}_s^{(L)} \otimes \mathbf{8}_v^{(R)}$ of SO(8) with $n_9 = 0$. These also have eight-dimensional masses $m_8^2 = w^2 \ R_9^2/\alpha'^2$.

In the limit $R_9 \to 0$ these last two sectors give us light fermion and p-form degrees of freedom. We shall discuss a T-dual interpretation of the spectrum shortly.

Full spectrum

Now we will list the full spectrum of states. We will also organize the ground states in each sector by their SO(8) quantum numbers, although SO(8) is broken by the compactification. The We have:

- Sector **A**: Bosonic states with $n_9 \in \mathbb{Z}$ and $w \in 2\mathbb{Z}$ and an even number of $\tilde{\theta}$ operators $N_{\tilde{\theta}} \in 2\mathbb{Z}$ acting on the $\mathbf{8}_v^{(L)} \otimes \mathbf{8}_v^{(R)}$ state of the fermion zero modes. The $\tilde{\theta}$'s are integrally moded.
- Sector **B**: Fermionic states with $n_9 \in \mathbb{Z}$ and $w \in 2\mathbb{Z}$ and an even number of $\tilde{\theta}$ operators $N_{\tilde{\theta}} \in 2\mathbb{Z}$ acting on the $\mathbf{8}_v^{(L)} \otimes \mathbf{8}_s^{(R)}$ state of the fermion zero modes. The $\tilde{\theta}$'s are integrally moded.
- Sector **C**: Fermionic states with $n_9 \in \mathbb{Z} + \frac{1}{2}$ and $w \in 2\mathbb{Z}$ and an odd number of $\tilde{\theta}$ operators $N_{\tilde{\theta}} \in 2\mathbb{Z} + 1$ acting on the $\mathbf{8}_v^{(L)} \otimes \mathbf{8}_v^{(R)}$ state of the fermion zero modes. The $\tilde{\theta}$'s are integrally moded.
- Sector **D**: Bosonic states with $n_9 \in \mathbb{Z} + \frac{1}{2}$ and $w \in 2\mathbb{Z}$ and an odd number of $\tilde{\theta}$ operators $N_{\tilde{\theta}} \in 2\mathbb{Z} + 1$ acting on the $\mathbf{8}_v^{(L)} \otimes \mathbf{8}_s^{(R)}$ state of the fermion zero modes. The $\tilde{\theta}$'s are integrally moded.

ground state phases in winding sectors such that the worldsheet CFT is modular invariant. This prescription was alluded to in [1] and will be discussed in more detail in future work.

- Sector **E**: Fermionic states with $n_9 \in \mathbb{Z}$ and $w \in 2\mathbb{Z}+1$ and an odd number of $\tilde{\theta}$ operators $N_{\tilde{\theta}} \in 2\mathbb{Z}+1$ acting on the $\mathbf{1}^{(L)} \otimes \mathbf{8}_v^{(R)}$ state of the fermion zero modes. The $\tilde{\theta}$'s are half-integrally moded.

- Sector **F**: Bosonic states with $n_9 \in \mathbb{Z}$ and $w \in 2\mathbb{Z}+1$ and an odd number of $\tilde{\theta}$ operators $N_{\tilde{\theta}} \in 2\mathbb{Z}+1$ acting on the $\mathbf{1}^{(L)} \otimes \mathbf{8}_s^{(R)}$ state of the fermion zero modes. The $\tilde{\theta}$'s are half-integrally moded.

- Sector **G**: Bosonic states with $n_9 \in \mathbb{Z}+\frac{1}{2}$ and $w \in 2\mathbb{Z}+1$ and an even number of $\tilde{\theta}$ operators $N_{\tilde{\theta}} \in 2\mathbb{Z}+1$ acting on the $\mathbf{1}^{(L)} \otimes \mathbf{8}_v^{(R)}$ state of the fermion zero modes. The $\tilde{\theta}$'s are half-integrally moded.

- Sector **H**: Fermionic states with $n_9 \in \mathbb{Z}+\frac{1}{2}$ and $w \in 2\mathbb{Z}+1$ and an even number of $\tilde{\theta}$ operators $N_{\tilde{\theta}} \in 2\mathbb{Z}+1$ acting on the $\mathbf{1}^{(L)} \otimes \mathbf{8}_s^{(R)}$ state of the fermion zero modes. The $\tilde{\theta}$'s are half-integrally moded.

In sectors $\mathbf{A}-\mathbf{F}$ the nine-dimensional mass of the lowest state is $m_9^2 = (\frac{|n_9|}{R_9} + \frac{R_9|w|}{\alpha'})^2$. Excited states are obtained by adding k units of oscillator weight to both L_0 and $\tilde{L}_0$ with $k \in \{0, 1, \cdots\}$. In every sector this gives a mass spectrum of $m_9^2 = (\frac{|n_9|}{R_9} + \frac{R_9|w|}{\alpha'})^2 + \frac{4k}{\alpha'}$.

In sectors **G** and **H** the nine-dimensional mass of the lowest state is $m_9^2 = (\frac{n_9}{R_9} + \frac{R_9 w}{\alpha'})^2$ if $n_9 w \geq -\frac{1}{2}$, and $m_9^2 = (\frac{n_9}{R_9} - \frac{R_9 w}{\alpha'})^2 - \frac{2}{\alpha'}$ if $n_9 w \leq -\frac{1}{2}$. Note that these two expressions agree when $n_9 w = -\frac{1}{2}$. So the spectrum in sectors $\mathbf{G}, \mathbf{H}$ is $m_9^2 = (\frac{n_9}{R_9} + \frac{R_9 w}{\alpha'})^2 + \frac{4k}{\alpha'}$ if $n_9 w \geq -\frac{1}{2}$ and $m_9^2 = (\frac{n_9}{R_9} - \frac{R_9 w}{\alpha'})^2 + \frac{4k-2}{\alpha'}$ if $n_9 w \leq -\frac{1}{2}$, for $k \in \{0, 1, \cdots\}$.

In the limit $R_9 \to \infty$ the states in sectors $\mathbf{E}-\mathbf{H}$ have $w \neq 0$, so they go to infinite mass. The states with $w = 0$ and fixed oscillator state form continua labelled by $p_9 = n_9/R_9$. These states live only in the sectors $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and **D**. Sector **A** contains the fields $G_{\mu\nu}, B_{\mu\nu}$, and Φ. Sector **D** contains the fields $C_\mu^{(\mathrm{RR})}$ and $C_{\mu\nu\sigma}^{(\mathrm{RR})}$. Sectors **B** and **C** respectively contain gravitini and dilatini of the two chiralities, though only those in sector **B** have massless modes in 9 dimensions for finite R_9, indicating that the Wilson line breaks the spacetime SUSY down to 16 supercharges in 9 dimensions.

The action of SUSY is particularly transparent here. Since the spacetime supercharges are generated entirely from right-moving currents, the spacetime supercharges must exchange sectors $\mathbf{A} \leftrightarrow \mathbf{B}$, sectors $\mathbf{C} \leftrightarrow \mathbf{D}$, sectors $\mathbf{E} \leftrightarrow \mathbf{F}$ and sectors $\mathbf{G} \leftrightarrow \mathbf{H}$. As for the multiplet structure for the massless states, we can infer it by state counting alone. There are 64 bosonic and 64 fermionic massless degrees of freedom in sectors **A** and **B** respectively. The bosons are the 27 states of the nine-dimensional graviton, the 21 states of the nine-dimensional NS-NS B-field, a KK vector G_{i9} and a B-vector B_{i9}, with 7 polarizations each, and two scalars Φ and G_{99}. The bosonic content of the gravitational multiplet of $\mathcal{N}=1$ SUSY in 8+1 dimensions is the graviton, a rank-two

antisymmetric tensor, a vector and a single real scalar. The remaining bosonic states – a vector and a real scalar – are the bosonic content of a massless vector multiplet of $\mathcal{N}=1$ SUSY in 9D.

So the low-energy theory at a generic radius R_9 contains only a gravitational multiplet and a massless vector multiplet. Both multiplets arise from sectors **A** and **B**. At a particular radius the massless spectrum will be enlarged by the appearance of extra vector multiplets, which make the continuous gauge symmetry nonabelian.

T-duality

In the limit $R_9 \to 0$ the states in sectors $\mathbf{C}, \mathbf{D}, \mathbf{G}, \mathbf{H}$ have $n_9 \neq 0$, so they become infinitely heavy and leave the spectrum. States with $n_9 = 0$ and fixed oscillator state form continua labeled by wR_9/α'. Sector **A** again contains a massless graviton, two scalars, and a rank-two tensor. Sector **B** still contains a massless gravitino and dilatino, even for finite $\tilde{R}_9$. Sectors **E** and **F** contain fermions and p-forms, respectively, which are massive for nonzero R_9 but form massless continua in the limit $R_9 \to 0$.

For small R_9 it is easiest to understand the spectrum in terms of a T-dual theory. The spectrum of winding and momentum states is invariant if we define

$$\tilde{n}_9 \equiv -w/2$$

$$\tilde{w} \equiv -2n_9$$

$$\tilde{R}_9 \equiv \alpha'/(2R_9)$$

which means

$$p_L \to +p_L \qquad\qquad p_R \to -p_R$$

The sectors, then are exchanged as

$$\mathbf{A} \leftrightarrow \mathbf{A} \qquad\qquad \mathbf{B} \leftrightarrow \mathbf{B}$$

$$\mathbf{C} \leftrightarrow \mathbf{E} \qquad\qquad \mathbf{D} \leftrightarrow \mathbf{F}$$

$$\mathbf{G} \leftrightarrow \mathbf{G} \qquad\qquad \mathbf{H} \leftrightarrow \mathbf{H}$$

It is convenient to switch to a T-dual set of local operators, using the change of variables $X^9_L \to +X^9_L$, $X^9_R \to -X^9_R$.

Let us now get a better sense of how the restored $SO(8)$ invariance looks in the limit $\tilde{R}_9 \to \infty$. To ensure that the right-moving spacetime supercurrent has a form with manifest $SO(8)$ invariance in the limit $\tilde{R}_9 \equiv \alpha'/(2R_9) \to \infty$, we must also change θ to $\Gamma^9\ \theta$. This changes the $SO(8)$ representation of the θ's from $\mathbf{8}_a$ to $\mathbf{8}_s$, which in turn changes the $SO(8)$ representation of their ground states from $\mathbf{8}_v^{(R)} \oplus \mathbf{8}_s^{(R)}$ to $\tilde{\mathbf{8}}_v^{(R)} \oplus \tilde{\mathbf{8}}_a^{(R)}$. (The tildes remind us that the new representation is really under a different $SO(8)$, which rotates the T-dual coordinate $\tilde{X}^9$ with the other X's, rather than the original X^9.)

So when R_9 goes to ∞, the gravitino states lie in an $\mathbf{8}_v^{(L)} \otimes \mathbf{8}_s^{(R)}$ in sector $\mathbf{B}$ and an $\mathbf{8}_a^{(L)} \otimes \mathbf{8}_v^{(R)}$ in sector $\mathbf{C}$. When R_9 is small, we make the replacements $\mathbf{8}_a^{(R)} \to \tilde{\mathbf{8}}_s^{(R)}$ and $\mathbf{8}_a^{(R)} \to \tilde{\mathbf{8}}_s^{(R)}$. The massless gravitini come from sectors $\mathbf{B}$ and $\mathbf{E}$. The $SO(8)$ representation of the massless states in sector $\mathbf{B}$ is

$$\mathbf{8}_v^{(L)} \otimes \mathbf{8}_s^{(R)} = \mathbf{8}_v^{(L)} \otimes \tilde{\mathbf{8}}_a^{(R)}.$$

The massless states in sector $\mathbf{E}$ transform as

$$\mathbf{8}_s^{(L)} \otimes \mathbf{8}_v^{(R)} = \mathbf{8}_s^{(L)} \otimes \tilde{\mathbf{8}}_v^{(R)}.$$

So the $SO(8)$ representations in the T-dual theory contain one gravitino of each chirality. Therefore the $R_9 \to 0$ limit of type IIA on an S^1 with a Wilson line for $(-1)^{F_{L_S}}$ is another ten-dimensional theory of type IIA, not a theory of type IIB.

An identical set of calculations and deductions shows that the $R_9 \to 0$ limit of type IIB on an S^1 with a Wilson line for $(-1)^{F_{L_S}}$ is another ten-dimensional theory of type IIB.

This is different from the situation of compactification on an ordinary circle, where taking the $R_9 \to 0$ limit exchanges type IIA with type IIB. In contrast to type II on an untwisted circle, type II on a circle with a Wilson line for $(-1)^{F_{L_S}}$ should have a special radius R_9 where the physics is self-dual, just as in the case of the bosonic string.

Enhanced gauge symmetry at the self-dual radius

When $R_9 = \sqrt{\frac{\alpha'}{2}}$ the theory is self-dual. In addition to the obvious discrete symmetry under $w \to -2n_9, n_9 \to -\frac{1}{2}w$, there is an enhanced continuous symmetry as well. In GS formalism the enhanced spin-1 currents are obscure and involve twist fields for the θ fermions, and we will not write them. Instead we will examine the spectrum and note that there are extra massless vector states which carry winding and momentum on the circle S^1.

By virtue of the mass formulae it is clear that sectors $\mathbf{C}, \mathbf{D}, \mathbf{E}, \mathbf{F}$ can never contain states with $m_9^2 = 0$. For arbitrary values of R_9 the sectors $\mathbf{A}, \mathbf{B}$ always contain states

with $m_9 = 0$, which are precisely those with $w = n_9 = 0$. The sectors $\mathbf{G}, \mathbf{H}$ can contain massless states if and only if R_9 is at the self-dual radius $R_9 = (\alpha'/2)^{\frac{1}{2}}$, and then only for $w = -2n_9 = \pm 1$. For those values, the oscillator ground states are level matched with $m^2 = 0$. They transform in the $\mathbf{8}_v$ of $SO(8)$, which decomposes as a vector and a scalar under the little group $SO(7)$ of a massless field in $8+1$ dimensions. Acting with a θ zero mode θ_0^a gives the eight massless polarizations of a minimal spinor of $SO(7)$. Together these fill out the states of a massless vector multiplet of the $\mathcal{N} = 1$ supersymmetric gauge theory in $D = 8 + 1$.

We get one such multiplet for each of the two possible values ± 1 for $w = -2n_9$, which labels the charge $q_L \equiv n_9 - \frac{1}{2}w$ under $U(1)_L$. The charge $q_R \equiv n_9 + \frac{1}{2}w$ under $U(1)_R$ is zero for both states. Consistency of the 9D effective field theory demands that the gauge group determined by the massless vectors be unitary. We have seen that it has dimension 3 and is nonabelian, so the gauge group coming from the left-moving currents must be $SU(2)$. We shall be able to examine the algebra of currents explicitly when we go to the RNS formalism.

Again by checking the mass formulae we can see that these are the only extra massless states we pick up for any value of R_9. So the enhanced gauge symmetry at the self-dual radius is $SU(2)_L \times U(1)_R$. Since SUSY is unbroken and the vacuum energy is zero, this tells us the supersymmetry cannot be gauged. At any rate, there is not enough SUSY to form a non-singlet representation of $SU(2)$. We can conclude from this that the graviphoton must be the gauge vector of $U(1)_R$, while the $SU(2)_L$ vectors must lie in vector multiplets under the $\mathcal{N} = 1$ spacetime SUSY. Counting of states supports the same conclusion.

Vertex operators

The construction of vertex operators in the light-cone GS description is somewhat involved and in the most familiar treatment [5] leans heavily on spacetime supersymmetry, which is partially broken by our Wilson line compactification. This will not impede us, since we will not be interested in the detailed form of the vertex operators. We will be interested only in the boundary conditions imposed by the vertex operators on the worldsheet fields $X^i, X^9, \theta, \tilde{\theta}$. While we only consider *states* with nonvanishing p_- in light-cone gauge, the simplest *vertex operators* to construct are exactly those with $p_- = 0$. As in [5], those shall be the ones we consider.

We will also be aided by a simple principle for understanding the meaning of the vertex operators of type $V_{bos}^{(L)} \otimes V_{bos}^{(R)}$ – just as in tbe bosonic string, they can be thought of as the operators defined by worldsheet Lagrangian perturbations induced by giving an expectation value to a certain string state. This will help us find the vertex operators corresponding to sectors $\mathbf{A}$ and $\mathbf{D}$.

- $V_{\mathbf{A}}$: Vertex operators for states of type **A** (except with vanishing p_-) are of the form $:\exp\{ik_iX^i\}:\ E_{(n_9,w)}\ V^{(L)}\ V^{(R)}$ with $n_9 \in \mathbb{Z}$ and $w \in 2\mathbb{Z}$. Here, $E_{(n_9,w)} \equiv: \exp\{ip_9^{(L)}X_L^9 + ip_9^{(R)}X_R^9\}:$ with $p_9^{(L)} \equiv \frac{n_9}{R_9} - \frac{wR_9}{\alpha'}$ and $p_9^{(R)} \equiv \frac{n_9}{R_9} + \frac{wR_9}{\alpha'}$. The internal pieces $V^{(L)}$ and $V^{(R)}$ can be taken to be ordinary bosonic operators with $\Delta^{(R)} - \Delta^{(L)} = wn$. The simplest representative of this sector is $w = n_9 = 0$, $k_i = 0$, $V^{(R)} = \partial_+X^j$ and $V^{(L)} = \partial_-X^i$. In the vicinity of such an operator the bosonic fields are single valued (they may have meromorphic singularities) and the fermionic fields $\theta, \tilde{\theta}$ are also single valued. In fact, at the level of boundary conditions for operators, $E_{(integer,even\ integer)}$, $:\exp\{ik_iX^i\}:$, $V^{(L)}$ and $V^{(R)}$ are all irrelevant. For purposes of boundary conditions on local fields, we can just take the operator to be 1. We shall write $V_{\mathbf{A}} \in [1]$.

- $V_{\mathbf{B}}$: One can obtain a state in sector **B** by acting with a right-moving fermion θ on a state in sector **A**. So we have $\mathbf{B} \in [\theta]$.

- $V_{\mathbf{C}}$: One can obtain a state in sector **C** by acting on a state in sector **A** with a left-moving fermion $\tilde{\theta}$ and an exponential $E_{(\frac{1}{2},0)} \equiv: \exp\{\pi i X^9/R_9\}:$ with half a unit of KK momentum. So $V_{\mathbf{C}} \in [E_{(1/2,0)}\ \tilde{\theta}]$.

- $V_{\mathbf{D}}$: One can obtain a state in sector **D** by acting with a right-moving fermion θ on a state in sector **C**. So we have $V_{\mathbf{D}} \in [E_{(1/2,0)}\ \theta\ \tilde{\theta}]$.

- $V_{\mathbf{E}}$: This sector gives rise to branch cuts in left-moving fermions $\tilde{\theta}$, so it includes a factor of a twist field for T. More precisely, let T be a ground state twist field, and S be a first excited twist field, with $T\ \tilde{\theta} \sim \bar{z}^{+\frac{1}{2}}S$. Then $V_{\mathbf{E}} \in [E_{(0,1)}\ S] = [E_{(0,1)}\ \tilde{\theta}\ T]$. The exponential $E_{(0,1)}$ is there because this sector has integral KK momentun and odd winding on the X^9 circle.

- $V_{\mathbf{F}}$: One can obtain a state in sector **F** by acting with a right-moving fermion on a state in sector **E**. So we have $V_{\mathbf{F}} \in [E_{(0,1)}\ \theta\ S] = [E_{(0,1)}\ \theta\ \tilde{\theta}\ T]$. Remember that the S and T fields are only spin fields for the $\tilde{\theta}$ fermions and not for the θ fermions.

- $V_{\mathbf{G}}$: One can obtain a state in sector **G** by acting on a state in sector **E** with a left-moving fermion $\tilde{\theta}$ and an exponential $E_{(\frac{1}{2},0)}$ with half a unit of KK momentum. So $V_{\mathbf{G}} \in [E_{(\frac{1}{2},1)}\ T]$.

- $V_{\mathbf{H}}$: One can obtain a state in sector **H** by acting with a right-moving fermion θ on a state in sector **G**. So we have $V_{\mathbf{H}} \in [E_{(\frac{1}{2},1)}\ \theta\ T]$.

For our purposes, the only meaningful distinction between the T and S fields is that the T's have $(-1)^{F_{L_S}} = +1$ and the S's have $(-1)^{F_{L_S}} = -1$.

We will only make use of these operators in order to derive constraints on the set of allowed D-branes in the theory.

2.2 D-branes and open strings in the Green-Schwarz formalism

The GS formalism in light-cone gauge can in a simple way only handle branes which are extended in one time and at least one spatial direction. So for now we will consider only Dp-branes, with $p = 1, \cdots, 9$. We can also treat zerobranes and D-instantons when we turn to the NSR description of our theories. Since we can always T-dual along an even number of trivial circles to return to our original theory, we need only consider branes which are extended along 7 or 8 of the X^i directions. The odd-dimensional branes, which are unstable at large R_9, do not have simple, manifestly $SO(8)$-covariant descriptions, but they will appear as T-duals of even dimensional branes in the limit $R_9 \to 0$.

D8-branes localized on S^1

The simplest case to consider will be that of the D8-brane. Since we are in type IIA string theory we expect this brane to be stable in the limit $R_9 \to \infty$. The boundary conditions[4] for the bosons X^i, X^9 are

$$\partial_n X^i = 0, \ \ i = 2, \cdots, 8 \ @\partial$$

$$X^9 = 0 \bmod 2\pi R_9 \ @\partial$$

In the limit $R_9 \to \infty$ this is just the standard D8-brane. The boundary condition for the scalars and fermions is

$$\partial_n X^i = 0 \ @\partial \qquad\qquad X^9 = 0 \ @\partial \qquad\qquad \theta_\alpha = \Gamma^9_{\alpha\dot\beta}\tilde\theta_{\dot\beta} \ @\partial$$

By acting on a timelike boundary with the vertex operators $E_{(0,2)}$ in sector **A** we can see we also need to include all boundary conditions

$$\partial_n X^i = 0 \ @\partial \qquad\qquad X^9 = 4\pi j R_9 \ @\partial \qquad\qquad \theta_\alpha = \Gamma^9_{\alpha\dot\beta}\tilde\theta_{\dot\beta} \ @\partial$$

for all $j \in \mathbb{Z}$. We can also act with the vertex operators $V_{\mathbf{F}}$, which change the sign of $\tilde\theta$ and change the value of X^9 by $2\pi(j+1)R_9$. This set of operators generates the boundary conditions

$$\partial_n X^i = 0 \ @\partial \qquad\qquad X^9 = 2\pi(2j+1)R_9 \ @\partial \qquad\qquad \theta_\alpha = -\Gamma^9_{\alpha\dot\beta}\tilde\theta_{\dot\beta} \ @\partial$$

[4]For brevity we use the notation '$@\partial$' to mean 'at the boundary'.

These are all the classical boundary conditions for dynamical fields forced by the action of the eight closed string sectors.

So the total set of allowed boundary conditions is

$$\partial_n X^i = 0 \ @\partial \qquad X^9 = 2\pi j R_9 \ @\partial \qquad \theta_\alpha = (-)^j \Gamma^9_{\alpha\dot\beta}\tilde\theta_{\dot\beta} \ @\partial$$

for any integer j.

The physically allowed open string states are then of the form

$$|w\rangle \equiv \sum_j \left| \begin{array}{c} @L\ \partial: \quad \theta = (-1)^j \Gamma^9 \tilde\theta,\ X^9 = 2\pi j R_9 \\ @R\ \partial: \quad \theta = (-1)^{j+w} \Gamma^9 \tilde\theta,\ X^9 = 2\pi (j+w) R_9 \end{array} \right\rangle,$$

which have winding number w. Now let us examine the spectrum in the possible open string sectors. The details of the spectrum depend on the value of w mod 2.

Open strings with even winding

The sum over j just implements the periodicity of X^9, meaning that the spectrum is the same for all j, and we take $j = 0$. When w is even, the left boundary $\sigma^1 = 0$ and right boundary $\sigma^1 = 2\pi r_{ws}$ have the same boundary condition $\tilde\theta = \Gamma^9\theta$ for the $\theta, \tilde\theta$ variables. For even w the mode expansion in the open string sector with winding w is

$$\theta_\alpha = \sum_{K=-\infty}^{\infty} \ b_{\alpha K} \exp\{iK(\sigma^1 + \sigma^0)/(2r_{ws})\}$$

$$\tilde\theta_{\dot\alpha} = \sum_{K=-\infty}^{\infty} \ \Gamma^9_{\dot\alpha\alpha} b_{\alpha K} \exp\{iK(-\sigma^1 + \sigma^0)/(2r_{ws})\}$$

The operators $b_{K\alpha} = C_{\alpha\beta} b^\dagger{}_{-K\alpha}$ create excitations with energy $\frac{K}{2r_{ws}}$ for $K < 0$ and destroy them for $K > 0$. The modes $b_{0\alpha}$ generate a Clifford algebra of which the ground states form a representation. Though $SO(8)$ is broken by the boundary conditions on the $\tilde\theta$ fermions, it is convenient to organize the fermion ground states as representations under the $SO(8)$ which acts on the θ fermions. The θ fermions transform in the $\mathbf{8}_a$ of $SO(8)$, so the fermion ground states transform in the representation $\mathbf{8}_v \oplus \mathbf{8}_s$. Under the unbroken $SO(7)$ this decomposes as $\mathbf{7}_v \oplus \mathbf{1} \oplus \mathbf{8}_{Maj.}$.

For $w = 0$ the content of the low-energy spectrum is a massless vector multiplet of $\mathcal{N} = 1$ gauge theory in 9 dimensions, just as on an ordinary D8-brane in type IIA string theory. In fact, it is immediate that the spectrum all open string sectors with even w is identical to what it would be for a D8-brane transverse a circle of radius R_9 with no Wilson line. This is because the only effect of the Wilson line on the open string sectors is to change the boundary conditions on the $\theta, \tilde\theta$ in sectors with odd winding.

Open strings with odd winding

In sectors of odd winding, the boundary condition is $\tilde{\theta} = \Gamma^9\theta$ on the left and $\tilde{\theta} = -\Gamma^9\theta$ on the right. So the mode expansion for the fermions is

$$\theta_\alpha = \sum_{K\in\mathbb{Z}+\frac{1}{2}} \; b_{\alpha K}\exp\{iK(\sigma^1+\sigma^0)/(2r_{ws})\}$$

$$\tilde{\theta}_{\dot\alpha} = \sum_{K\in\mathbb{Z}+\frac{1}{2}} \; \Gamma^9_{\dot\alpha\alpha} b_{\alpha K}\exp\{iK(-\sigma^1+\sigma^0)/(2r_{ws})\}$$

Now the frequencies of the eight sets of fermionic oscillators are $\frac{2K+1}{4r_{ws}}$ with K in $\mathbb{Z}$. The eight bosonic oscillators have frequencies $\frac{K}{2r_{ws}}$ with $K\in\mathbb{Z}$ as usual. The total ground state energy contributed by all the oscillators together is $-\frac{1}{4r_{ws}}$. So the mass spectrum for odd winding w is

$$m_9^2 = \frac{w^2R_9^2}{\alpha'^2} - \frac{1}{\alpha'} + \frac{K^{tot}}{\alpha'}$$

where K^{tot} is the total (integer) number of units of oscillator energy. Moreover, K^{tot} is even for bosonic states and odd for fermionic states, so we can see that the supersymmetric Bose-Fermi degeneracy is broken in sectors of odd winding. In fact for w odd and $R_9 < \sqrt{\alpha'}/|w|$ the lowest state of the open string is a tachyon!

The first excited states come from acting with $b_{\alpha-1}$ on the vacuum. These states transform as an $\mathbf{8}_a$ under the SO(8) which rotates the θ's, and as an $\mathbf{8}_{Maj}$ under the unbroken $SO(7)$. The mass of these fermions is given by $m^2 = w^2R_9^2/\alpha'^2$, so they become light in the $R_9\to 0$ limit. We shall now examine that limit.

T-dual of the D8-brane

Starting at large radius and lowering R_9, the eightbrane is stable until we reach $R_9=\sqrt{\alpha'}$, which is $\sqrt{2}$ times the self-dual radius. We can continue lowering R_9 past the point of instability, even past the self-dual radius. Since our brane is localized in the X^9 direction, it should be extended in the T-dual coordinate $\tilde{X}^9$, so clearly going beyond the T-dual radius gives us a different brane, though the closed string background is of the same type with which we started.

To get some idea of what the spectrum of the T-dual brane might look like, take R_9 to zero and consider modes with fixed $\tilde{p}_9 \equiv \tilde{n}_9/\tilde{R}_9 = 2\tilde{n}_9R_9/\alpha' = wR_9/\alpha'$. In this limit the winding modes form a continuum, and the distinction between even and odd winding disappears – we have both sets of states at every value of $\tilde{p}_9\in\mathbb{R}$.

From the spectrum with $w\in 2\mathbb{Z}$, we get a set of massless bosons which transform under the $SO(8)$ of the θ's as $\mathbf{8}_v$. Therefore it must also transform as an $\mathbf{8}_v$ under the new $SO(8)$ of the T-dual theory. The massless fermions of the even winding sector transform as an $\mathbf{8}_s$ under $SO(8)_\theta$, so they transform as an $\tilde{\mathbf{8}}_a$ under the $SO(8)$ of the

T-dual theory, by the same argument we made in the section on T-duality of the closed string theory.

The odd-winding sectors contribute a real tachyon with mass $m_9^2 = p_9^2 - \frac{1}{\alpha'}$, which corresponds to a field with $m_{10}^2 = -\frac{1}{\alpha'}$, which is a tachyon in the ten dimensional sense. We saw that the first excited states of the odd-winding sectors are a set of fermions with $m_9^2 = p_9^2$, so in the ten-dimensional sense they are massless. Under $SO(8)_\theta$ they transform in the $\mathbf{8}_a$, so under the SO(8) of the T-dual theory they transform in the $\tilde{\mathbf{8}}_s$.

We have found that the spectrum which is massless in the ten-dimensional sense on the T-dual side is given by a vector and a set of fermions which transform under the little group as $\tilde{\mathbf{8}}_a \oplus \tilde{\mathbf{8}}_s$, corresponding to a non-chiral Majorana fermion of $SO(9,1)$. There is also a real open string tachyon of mass-squared $-\frac{1}{\alpha'}$. At the massless and tachyonic level, we have the same open string spectrum as that of a non-BPS ninebrane in type IIA in 10 dimensions. So the natural conjecture is that the $R_9 \to 0$ limit of a charged D8-brane transverse to an S^1 with a Wilson line for $(-1)^{F_{L_S}}$ gives an uncharged, non-BPS D9-brane in type IIA.

Some basic checks support this idea. The simplest is the agreement of the 8+1 dimensional energy densities of the two kinds of effective eightbrane. Under T-duality on a circle, the dilaton must always transform such that

$$\frac{R_9}{g_s^2} = \frac{\tilde{R}_9}{{g_s'}^2}.$$

Since our T-duality transformation is

$$\tilde{R}_9 = \frac{\alpha'}{2R_9},$$

we have

$$\frac{g_s'}{g_s} = \sqrt{\frac{\alpha'}{2}}\ R_9^{-1} = \sqrt{\frac{2}{\alpha'}}\ \tilde{R}_9$$

The tension of a D8-brane at string coupling g_s is given [2] by

$$T_{\rm D8}^{(g_s)} = \frac{1}{g_s\ (2\pi)^8\ \alpha'^{9/2}}$$

Using the relation we just derived, we can rewrite this expression in terms of the T-dual coupling and radius:

$$T_{\rm D8}^{(g_s)} = \sqrt{2}\ \frac{\tilde{R}_9}{g_s'\ (2\pi)^8\ \alpha'^5} = \sqrt{2}\ \frac{2\pi\tilde{R}_9}{g_s'\ (2\pi)^9\ \alpha'^5}$$

$$= \sqrt{2}\ T_{\rm D9}^{(g_s')}\ (2\pi\tilde{R}_9)$$

We have defined the quantity $T_{\rm D9}^{(g_s')}$ to be $1/(g_s'\ (2\pi)^9\ \alpha'^5)$, which is the tension of a standard BPS D9-brane in type IIB string theory, at string coupling g_s'. If this

were equal to the tension of a non-BPS D9-brane in type IIA string theory our T-duality conjecture would be falsified. However it has been observed [3] that the tension $T_{\rm non-BPS\ D9}$ of a non-BPS ninebrane at a given string coupling is larger than that of a BPS D9-brane by a factor of $\sqrt{2}$. As a result we find

$$T_{\rm D8}^{(g_s)} = \sqrt{2}\, T_{\rm D9}^{(g_s{}')}\ (2\pi\tilde{R}_9) = T_{\rm non-BPS\ D9}^{(g_s{}')}\ (2\pi\tilde{R}_9)$$

$$T_{\rm non-BPS\ D9}^{(g_s{}')}\ \tilde{V}_9,$$

which agrees with our T-duality proposal for the D8-brane. Note that this agreement depends on the extra factor of 2 modifying the usual T-duality relation between R_9 and $\tilde{R}_9$.

In type IIA a non-BPS ninebrane preserves the symmetry $(-1)^{F_{L_S}}$ and the real open string tachyon on the ninebrane is odd under this symmetry. In the T-dual picture, then, when $\tilde{R}_9$ is finite, the open string tachyon should be antiperiodic around the $\tilde{X}^9$ direction because there is a Wilson line for $(-1)^{F_{L_S}}$ in the dual picture as well as the original picture. So only tachyon modes with half-integral momentum $\tilde{n}_9 \in \mathbb{Z} + \frac{1}{2}$ should exist. Since $\tilde{n}_9 = w/2$, this agrees perfectly with the fact that in the original eightbrane picture, the tachyonic modes exist only for odd winding $w \in 2\mathbb{Z} + 1$.

It would be good to check the T-duality conjecture in more detail, in particular to check directly that the full $SO(9,1)$ or even just $SO(8)$ is restored in the T-dual theory in the limit $R_9 \to 0$. We will soon study these open string theories in the RNS framework, where Lorentz invariance in all directions is more manifest.

Endpoint of tachyon condensation

Now we take a slight detour into a question to which we can offer only a conjectural answer. The question is: what is the endpoint of open string tachyon condensation for the D8-brane on a circle, when $R_9 < \sqrt{\alpha'}$? The default hypothesis might be that tachyon condensation should lead to the closed string vacuum, since there are no absolutely stable, charged branes to which it is possible to decay.

We claim that the actual situation is more interesting, with an endpoint for the instabilities which depends on the value of R_9:

- For $\sqrt{\alpha'} < R_9$ the brane may be absolutely stable.
- For $\sqrt{\alpha'}/2 < R_9 < \sqrt{\alpha'}$ it is possible that the brane may decay completely, since we know of no stable branes of any kind in this range. In particular, when $R_9 > \sqrt{\alpha'}/\sqrt{2}$, the D8-brane is the lightest of the effective eightbrane in 9 dimensions which we study here. Other than the closed string vacuum, there is

no obvious lighter state to which it can decay. See the comments below, however, for discussion of a subtlety relating to discrete charge conservation.

- When $R_9 < \sqrt{\alpha'}/\sqrt{2}$, the non-BPS D9-brane in the original IIA theory, wrapped on the S^1, is lighter than the D8-brane. It is possible that the non-BPS ninebrane could be the endpoint of the decay of the D8-brane, though since it has its own instability this seems unlikely without fine-tuning or a symmetry.

- For $R_9 < \sqrt{\alpha'}/2$ the D9-brane wrapped on S^1 in the original IIA theory becomes perturbatively stable. We would now like to argue that for $R_9\sqrt{\alpha'}/2$ the endpoint of open string tachyon condensation on the D8-brane is actually a non-BPS D9-brane wrapping the circle.

To illustrate this last decay scenario, go to the T-dual frame, where the D8-brane becomes a $\widetilde{\mathrm{D9}}$-brane wrapping the $\widetilde{\mathrm{S}}^1$. The lightest winding tachyon in the original theory becomes the lightest tachyon KK mode in the dual theory – the tachyon mode with $\tilde{n}_9 = \frac{1}{2}$. The profile of this mode is $\sin(\tilde{x}_9/(2\tilde{R}_9))$. Assuming $\tilde{R}_9$ to be macroscopically large, we can treat the $\widetilde{\mathrm{D9}}$-brane as noncompact, and the profile for the tachyon as linear, with a zero at the origin $\tilde{x}^9 = 0$. The endpoint of such inhomogenous tachyon condensation is believed to be a charged D8-brane localized at the origin $\tilde{x}_9 = 0$ [3]. Returning to the original IIA theory, this becomes a non-BPS D9-brane wrapping the circle. Since the circle has very small radius R_9, this transition is allowed energetically. And there are no perturbative instabilities of the endpoint of the condensation for $R_9 < \frac{1}{2}\sqrt{\alpha'}$, so the a small perturbation to final stages of the condensation will not destroy the ninebrane.

The range $\sqrt{\alpha'}/2 < R_9 < \sqrt{\alpha'}$ may display a more subtle behavior than simple decay to the closed string vacuum, due to a possible mechanism which would rely on a $\mathbb{Z}_2$-valued brane charge.

To motivate the existence of such a charge, consider the theory at large radius R_9. There, the eightbrane has no perturbative instability, but there are virtual processes involving topologically nontrivial Euclidean eightbranes which can change the D8 into an anti-D8. For instance, the D8 can move around the circle by an amount $2\pi R_9$; when it comes back to itself the sign of the RR charge has been reversed, and brane charge has been violated mod 2. Alternately, a D8-$\overline{\mathrm{D8}}$ pair can nucleate from the vacuum, with one travelling around the x^9-circle while the other stays put. One ends up with two D8's or two $\overline{\mathrm{D8}}$'s. For large R_9, this type of process, with an Euclidean eightbrane worldvolume winding the circle, is the only type of process by which eightbrane charge can change.

We can see, then, that the large-radius effective theory has a $\mathbb{Z}_2$-valued conservation law for eightbrane charge. If one goes a step further and assumes that stringy effects preserve this discrete conservation law, then there is a discrete charge which should be conserved mod 2 in any quantum process. The presence of winding tachyons is a stringy effect not described in the effective theory of branes and gauge quanta at large radius; their presence completely changes the mechanism which violates $\mathbb{Z}$-valued brane charge, relative to the brane-loop mechanism of the large-radius effective field theory. But if $\mathbb{Z}_2$-valued charge conservation survives into the stringy régime, then the endpoint of D8 decay could not be the closed string vacuum. Since there are no 'simple' branes with eight extended dimensions which are lighter than the original D8 in this range, the endpoint would have to be a described by an interacting boundary CFT, possibly corresponding to an inhomogeneous phase of the unstable ninebrane.

2.3 Closed strings in the Ramond-Neveu-Schwarz formalism

Having described the spectrum in the GS formalism, we will now give a description of the same theory in the RNS framework. This will make Lorentz invariance and T-duality more transparent. It will also allow us to discuss branes which are localized in all the noncompact spatial directions. Finally, the RNS formalism facilitates the construction of vertex operators by reducing it to the state-operator correspondence in a two dimensional CFT.

Fast review of the RNS approach

Our CFT is a free one, based on ten worldsheet fields X^μ and their left- and right-moving superpartners $\tilde{\psi}^\mu, \psi^\mu$. The theory has a $(1,1)$ worldsheet supersymmetry which commutes with the $SO(9,1)$ and exchanges the X^μ's with their superpartners in the usual way. The $SO(9,1)$ is broken only by the compactification of the X^9 direction.

In addition there are left- and right-moving $\tilde{b}, \tilde{c}$ and b, c ghosts and also superghosts $\tilde{\beta}, \tilde{\gamma}$ and β, γ. On the cylinder the only effect of the ghosts and superghosts is to contribute $-\frac{1}{2}$ to the ground state weight of NS sectors and $-\frac{5}{8}$ to the ground state weight of R sectors. Physical states are superconformal primaries of weight zero, which for us will be equivalent to superconformal matter primaries of weight $+\frac{1}{2}$ in NS sectors and $+\frac{5}{8}$ in R sectors.

We will always use the terms R and NS to label sectors strictly according to the periodicity of the supercurrent in a given sector, rather than to refer to boundary conditions for some particular free fermion. The notion of 'natural periodicity' – periodic for bosons and antiperiodic for fermions – simplifies the computation of ground state

weights in a given sector; real fields (bosonic or fermionic) with their natural periodicity contribute 0 to the ground state weight, and real fields with unnatural periodicity contribute $+\frac{1}{16}$.

In RNS framework the consistency conditions of a string theory are particularly transparent: one needs only modular invariance, tadpole cancellation, and closure and single-valuedness of the algebra of vertex operators. Modular invariance will follow from level matching and the inclusion of twisted sectors; tadpole cancellation is a result of the $\mathcal{N}=1$ spacetime supersymmetry. We will see these two features of the theory in this section, and we will defer the discussion of vertex operators to the next section. Taken together, these properties will demonstrate the consistency of our theory.

Closed string states

Since we have labelled the sectors **A** through **H**, we will be able to describe the states sector by sector in the RNS formalism. At the level of states on the cylinder this is mostly straightforward. The only subtlety is that the GSO projection differs from the usual one in certain sectors carrying momentum and winding. This is necessary to ensure level matching, and as we shall see later it is also necessary in order to make the algebra of vertex operators close with single-valued coefficient functions in the OPE.

The superghosts contribute a minus sign to the GSO phases $(-1)^{F_{L_W}}$ and $(-1)^{F_{R_W}}$ in NS sectors. (By F_{L,R_W} we mean left- and right-moving *worldsheet* fermion number.) So the GSO projection for the matter states in NS sectors is always opposite that for the states in the full theory with ghosts and superghosts. In the usual type IIA superstring the GSO projection in the full theory is + in NS sectors, + for right-moving R sectors and - for left-moving R sectors. We shall see that this is different in our theory – the constraints of level matching demand that modify all left-moving GSO projections by a sign of $(-1)^w$. For convenience we will sometimes ignore the ghosts and refer directly to the GSO projection in the matter sector, which for NS states is opposite that in the full theory with ghosts. We will try to observe this difference by distinguishing between the 'full GSO' and 'matter GSO', as well as between 'matter vertex operators' and full vertex operators with ghost and superghost dressing.

- Sector **A** : These are the NS-NS states with integral momentum and even winding: $n_9 \in \mathbb{Z}$ and $w \in 2\mathbb{Z}$. The GSO projection is the usual one; the full GSO is $+/+$. Therefore the matter GSO is $-/-$. For $n = w = 0$ the oscillator vacuum of the matter theory has weight $(0,0)$, so the lowest states satisfying the GSO projection are $\tilde{\psi}^{\mu}_{-\frac{1}{2}}\psi^{\nu}_{-\frac{1}{2}}\left|0\right\rangle$. These are weight $(+\frac{1}{2},+\frac{1}{2})$, so the momentum dressing must have weight zero. The polarizations of the massless fields are transverse and are defined up to a null-state redundancy, so the representations under the $SO(7)$

little group are $\Lambda^2\mathbf{7}\oplus\mathrm{Sym}^2\mathbf{7}\oplus\mathbf{7}\oplus\mathbf{7}\oplus\mathbf{1}$, which can be viewed as the decomposition of the SO(8) representation $\mathbf{8}_v \otimes \mathbf{8}_v$ after the SO(8) is broken to SO(7) by the compactification. Therefore the states with $m_9^2 = 0$, can be identified with the nine-dimensional graviton, dilaton, B-field, KK and B vectors, and the modulus G_{99}

- Sector **B**: These are the NS-R states with $n_9 \in \mathbb{Z}$ and $w \in 2\mathbb{Z}$. The matter GSO projection is $-/+$. The states in this sector are the superpartners of the states in sector **A**. The massless states are the dimensional reduction of one set of the ten-dimensional dilatino $\psi_{\dot\alpha}^{(R)}$ and gravitino $\Psi_{\alpha}^{\mu(R)}$. The little group representation of the massless fermions is the decomposition of $\mathbf{8}_v \otimes \mathbf{8}_s$ under the umbroken SO(7).

- Sector **C** : These are the R-NS states with $n_9 \in \mathbb{Z} + \frac{1}{2}, w \in 2\mathbb{Z}$. The full GSO is $-/+$ and the matter GSO is $-/-$. This sector contains other set of ten-dimensional gravitini and dilatini $\psi_{\alpha}^{(L)}, \Psi_{\dot\alpha}^{\mu(L)}$. These fields do not give rise to nine-dimensional massless fields because they are antiperiodic on the circle due to the Wilson line. Indeed, tsector **C** has no states which have m_9^2.

- Sector **D** : These are the R-R states with $n_9 \in \mathbb{Z} + \frac{1}{2}, w \in 2\mathbb{Z}$. The full GSO is $-/-$. These states are the superpartners of states in sector **C**. The states with $w = 0$ represent modes of the ten-dimensional RR vector C_μ and three-form $C_{\mu\nu\sigma}$ which are antiperiodic on the S^1. Note that since the physical polarizations of the RR forms are antiperiodic, it follows from SO(8,1) invariance that the timelike components C_0 and C_{0ij} must be as well. This gives a simple way to understand how branes such as the D8-brane can be unstable in this compactification: the lagrange multiplier fields which enforce charge conservation are zero modes of the p-form gauge fields, and these are projected out by the Wilson line.

- Sector **E** : These are the strings in the R/NS sector with integral momentum $n_9 \in \mathbb{Z}$ and odd winding $w \in 2\mathbb{Z} + 1$. The full GSO is $+/+$ and the matter GSO is $+/-$. These fermionic states are massive for any nonzero value of $R_9 = \alpha'/2\tilde{R}_9$, but they contain a massless continuum in the limit $\tilde{R}_9 \to 0$ which we interpret as a second set of ten-dimensional gravitini and dilatini in the 10-dimensional T-dual theory. In the RNS approach the SO(9,1) in the $\tilde{R}_9 \to \infty$ limit is transparent, because T-duality is simply the change of variables $X_R^9 \to -X_R^9$ along with $\psi^9 \to -\psi^9$. Since the spinor representntations are generated by the Clifford algebras of products of worldsheet fermion zero modes, it follows that T-duality reverses the $SO(9,1)$ chirality of right-moving R sectors and preserves the $SO(9,1)$ chirality of left-moving R sectors.

- Sector **F** : These are the strings in the R/R sector with integral momentum $n_9 \in \mathbb{Z}$ and odd winding $w \in 2\mathbb{Z}+1$. They are the superpartners of states in sector **E**. The full GSO is $+/+$. This set of states contains a tower which becomes a massless continuum of RR fields in the limit $\tilde{R}_9 \to \infty$. Since T-duality reverses the sign of the GSO projection in right-moving R sectors, the GSO projection of this sector in T-dual variables is $+/-$, so the RR forms of the dual IIA theory are a 1-form and 3-form, just as in the original IIA theory.

- Sector **G** : These are strings in the NS/NS sector with half-integral momentum $n_9 \in \mathbb{Z}+\frac{1}{2}$ and odd winding $w \in 2\mathbb{Z}+1$. The level mismatch due to the winding and momentum is $L_0 - \tilde{L}_0$ is $n_9 w$ mod 1, and it equals $\frac{1}{2}$ in this sector. It must be cancelled with oscillator energies, so the GSO projection must be $+/-$ or $-/+$ in order to have level-matched states. Closure of the algebra of vertex operators (discussed in a later section) shows that the consistent choice is $-/+$ for the full GSO, which gives $+/-$ for the matter GSO. This choice of GSO projection is a bit unusual, since it means that there are physical NS/NS vertex operators containing odd numbers of worldsheet fermions. However there is no inconsistency, since the half-integral spin of these operators is offset by the contribution coming from the exponential $:\exp\{ik_L X_L + ik_R X_R\}:$, which also has half-integral spin. This sector contains only massive states at generic radii, but at the self-dual radius $R_9 = \sqrt{\frac{\alpha'}{2}}$ it contains two extra massless gauge fields and a complex massless scalar. The gauge fields are the states $e_i \psi^i_{-\frac{1}{2}} \left|0;\ n_9 = \pm\frac{1}{2}, w = \mp 1\right\rangle$ and the massless scalars are the states $\psi^9_{-\frac{1}{2}} \left|0;\ n_9 = \pm\frac{1}{2}, w = \mp 1\right\rangle$. These states are the off-diagonal components of the $SU(2)$ vector boson and Hermitean adjoint scalar at the self-dual radius. They are the bosonic fields of the 8+1 dimensional $\mathcal{N}=1$ vector multiplet.

- Sector **H**: These are the strings in the NS/R sector with $n_9 \in \mathbb{Z}+\frac{1}{2}$ and odd winding $w \in 2\mathbb{Z}+1$. The full GSO projection for these states is $-/+$ and the matter GSO is $+/+$. They are the superpartners of the states in sector **G**. Generically massive, at the self-dual radius they contain the off-diagonal fermionic components of the $SU(2)$ vector multiplet of 8+1 dimensional $\mathcal{N}=1$ SUSY.

Closed string vertex operators for the $(-1)^{F_{Ls}}$ *Wilson line in RNS formalism*

To find the -1 or $-\frac{1}{2}$ picture vertex operator corresponding to a given superconformal primary matter state, simply apply the state-operator correspondence in the matter theory. Then to find the 0 picture matter operators for NS sectors, act with the raising operator $G_{-\frac{1}{2}}$ (or $\tilde{G}_{-\frac{1}{2}}$ for left movers). This is equivalent to applying a

picture-changing operator as described in [10]. We will describe the properties of the matter pieces of the vertex operators only; including the ghost and superghost factors in various pictures works as usual. Letting the indices $M, N, \cdots$ run from 0 to 8 we list the sectors.

- $V_{\mathbf{A}}$: This sector has $(-1,-1)$ picture matter vertex operators $\mathrm{NS}_-/\mathrm{NS}_-$ with $w, 2n_9 \in 2\mathbb{Z}$. For instance $\tilde{\psi}^\mu \psi^\nu$ are the matter vertex operators for the lowest massless states. The corresponding $(0,0)$ picutre operators are $\mathrm{NS}_+/\mathrm{NS}_+$ states, for instance $\partial_- X^\mu \partial_+ X^\nu$. These correspond transparently to the 9-dimensional metric G_{MN}, B-field B_{MN}, dilaton Φ KK vectors G_{M9}, B_{M9} and modulus G_{99}.

- $V_{\mathbf{B}}$: These $(-1,-\frac{1}{2})$ picture matter operators are in the $\mathrm{NS}_-/\mathrm{R}_+$ sector with $w, 2n_9 \in 2\mathbb{Z}$. The $(0,-\frac{1}{2})$ picture matter vertex operators are in the $\mathrm{NS}_+/\mathrm{R}_+$ sector. For instance, the operators include the massless 9D gravitini/dilatini matter vertex operators, $\tilde{\psi}^\mu \Sigma_\alpha$ in the $(-1,-\frac{1}{2})$ picture, $\partial_- X^\mu \Sigma_\alpha + o(k_\mu)$ in the $(0,-\frac{1}{2})$ picture.

- $V_{\mathbf{C}}$: $\mathrm{R}_-/\mathrm{NS}_-$ or $\mathrm{R}_-/\mathrm{NS}_+$ matter vertex operators in the $(-\frac{1}{2},-1)$ and $(-\frac{1}{2},0)$ pictures, respectively, with $n_9 \in \mathbb{Z}+\frac{1}{2}$ and $w \in 2\mathbb{Z}$. A typical matter vertex operator is $:\exp\{ik_M X^M\}:\ E_{(\frac{1}{2},0)}\ \tilde{\Sigma}_{\dot{\alpha}}\ (e_\mu \psi^\mu)$ in the $(-\frac{1}{2},-1)$ picture. This is a massive spin-3/2 field in 9 dimensions.

- $V_{\mathbf{D}}$: $\mathrm{R}_-/\mathrm{R}_-$ matter vertex operators with $n_9 \in \mathbb{Z}+\frac{1}{2}$ and $w \in 2\mathbb{Z}$. A typical matter vertex operator is $:\exp\{ik_M X^M\}:\ \ E_{(\frac{1}{2},0)}\ \tilde{\Sigma}_{\dot{\alpha}}\ {\Sigma'}_{\dot{\alpha}}$. This is a set of massive p-form fields in 9 dimensions. Taking the OPE with Σ_α implements SUSY transformations between sectors **C** and **D**.

- $V_{\mathbf{E}}$: These fermionic strings are R/NS sectors with integral momentum n_9 and odd winding w. The $(-\frac{1}{2},-1)$ picture matter vertex operators have GSO projection +/- and the $(-\frac{1}{2},0)$ picture matter vertex operators have GSO projection +/+. A typical operator is $:\exp\{ik_M X^M\}:\ \ E_{(0,1)}\ \tilde{\Sigma}_a{}'\ (e_\mu \psi^\mu)$ in the $(-\frac{1}{2},-1)$ picture.

- $V_{\mathbf{F}}$: These bosonic string modes are R/R sectors with integral momentum n_9 and odd winding w. The matter vertex operators have GSO projection +/+. A typical operator is $:\exp\{ik_M X^M\}:\ \ E_{(0,1)}\ \tilde{\Sigma}_a{}'\ \Sigma_\alpha$.

- $V_{\mathbf{G}}$: The matter vertex operators for these bosonic strings have $n_9 \in \mathbb{Z}+\frac{1}{2}$ and $w \in \mathbb{Z}$. They are in the $\mathrm{NS}_+/\mathrm{NS}_-$ sector in the $(-1,-1)$ picture, and the in the $\mathrm{NS}_-/\mathrm{NS}_+$ sector in the $(0,0)$ picture. A typical $(-1,-1)$ picture matter vertex operator is $:\exp\{ik_M X^M\}:\ E_{(\frac{1}{2},-1)}\ \psi^\mu$. Note the peculiarity that this NS/NS

Sector	n mod 1	w mod 2	b.c. and full GSO
A	0	0	$\mathrm{NS}_+/\mathrm{NS}_+$
B	0	0	$\mathrm{NS}_+/\mathrm{R}_+$
C	$\frac{1}{2}$	0	$\mathrm{R}_-/\mathrm{NS}_+$
D	$\frac{1}{2}$	0	$\mathrm{R}_-/\mathrm{R}_+$
E	$\frac{1}{2}$	1	$\mathrm{R}_+/\mathrm{NS}_+$
F	$\frac{1}{2}$	1	$\mathrm{R}_+/\mathrm{R}_+$
G	0	1	$\mathrm{NS}_-/\mathrm{NS}_+$
H	0	1	$\mathrm{NS}_-/\mathrm{R}_+$

Table 1: Closed string sectors in the background of a Wilson line for $(-1)^{F_{L_S}}$.

	A	B	C	D	E	F	G	H
A	A	B	C	D	E	F	G	H
B	B	A	D	C	F	E	H	G
C	C	D	A	B	G	H	E	F
D	D	C	B	A	H	G	F	E
E	E	F	G	H	A	B	C	D
F	F	E	H	G	B	A	D	C
G	G	H	E	F	C	D	A	B
H	H	G	F	E	D	C	B	A

Table 2: Multiplication rules for closed string vertex operators in the background of a Wilson line for $(-1)^{F_{L_S}}$.

vertex operator has odd worldsheet fermion number. Yet it has zero spin on account of the spin contributed by $E_{(\frac{1}{2},-1)}$.

- $V_{\mathbf{H}}$: The matter pieces of the vertex operators for these fermionic strings have $n_9 \in \mathbb{Z} + \frac{1}{2}$ and $w \in \mathbb{Z}$. They are in the $\mathrm{NS}_+/\mathrm{R}_+$ sector in the $(-1, -\frac{1}{2})$ picture, and the in the $\mathrm{NS}_-/\mathrm{R}_+$ sector in the $(0, -\frac{1}{2})$ picture. A typical $(-1, -\frac{1}{2})$ matter vertex operator is $: \exp\{ik_M X^M\} : E_{(\frac{1}{2},-1)}\, \Sigma_\alpha$.

We summarize our set of sectors in table (1). One can then check directly the closure of the algebra of vertex operators, and we display the multiplication table in table (2). We display a separate list, table (3), summarizing the massless spectrum for $R_9 \neq 0, \infty$.

Field	Vertex Op.	Sector	Massless in 9D?
$G_{\mu\nu}, \Phi$	$\tilde{c}c\ \exp\{-\tilde{\phi} - \phi + ik_P X^P\}\tilde{\psi}_{(\mu}\psi_{\nu)}$	A	always
$B_{\mu\nu}$	$\tilde{c}c\ \exp\{-\tilde{\phi} - \phi + ik_P X^P\}\tilde{\psi}_{[\mu}\psi_{\nu]}$	A	always
Ψ^μ_α	$\tilde{c}c\ \exp\{-\tilde{\phi} - \phi/2 + ik_P X^P\}\tilde{\psi}^\mu\Sigma_\alpha$	B	always
$\tilde{A}^1_M \pm i\tilde{A}^2_M$	$\tilde{c}c\ \exp\{-\tilde{\phi} - \phi + ik_P X^P\}E_{(\pm\frac{1}{2},\mp 1)}\psi^M$	G	at $R_9 = \sqrt{\frac{\alpha'}{2}}$
$\phi^1 \pm i\phi^2$	$\tilde{c}c\ \exp\{-\tilde{\phi} - \phi + ik_P X^P\}E_{(\pm\frac{1}{2},\mp 1)}\psi^9$	G	at $R_9 = \sqrt{\frac{\alpha'}{2}}$
$\Upsilon^1_\alpha \pm i\Upsilon^2_\alpha$	$\tilde{c}c\ \exp\{-\tilde{\phi} - \phi/2 + ik_P X^P\}E_{(\pm\frac{1}{2},\mp 1)}\Sigma_\alpha$	H	at $R_9 = \sqrt{\frac{\alpha'}{2}}$

Table 3: Massless sectors for finite $R_9, \tilde{R}_9$ with a Wilson line for $(-1)^{F_{L_S}}$.

Open string vertex operators in the RNS framework

The vertex operators for open strings in the RNS framework are similar to the ones for closed strings. In introducing the vertex operators, it is useful to define the boundary spin fields $\sigma_\alpha, {\sigma'}_{\dot\alpha}$. Ordinarily these are simply taken to be boundary values of the bulk spin fields $\Sigma, \tilde{\Sigma}$. Since our branes do not preserve spacetime supersymmetry, there can be no linear relation between the operators Σ and $\tilde{\Sigma}$ at the boundary which holds as an operator identity across all sectors. Therefore it is worth paying a bit more attention than usual to the properties of the $\sigma_\alpha, {\sigma'}_{\dot\alpha}$.

We should define the $\sigma_\alpha, {\sigma'}_{\dot\alpha}$ to be the fields which change the boundary conditions of worldsheet fermions from $\psi = \pm\tilde{\psi}$ to $\psi = \mp\tilde{\psi}$. To do this, the boundary spin field must create an equal and opposite phase in the fermions as they pass it on the right – a phase of $\exp\{+\pi i/2\}$ for the ψ's and $\exp\{-\pi i/2\}$ for the $\tilde{\psi}$'s, as one moves from $z \in -i\mathbb{R}^+$ to $z \in +i\mathbb{R}^+$, through a contour with Re $z > 0$. Therefore the boundary spin fields should have OPE's such as:

$$\sigma_\alpha(0)\ \psi^\mu(z) \sim z^{-\frac{1}{2}}\ \Gamma^\mu_{\alpha\dot\alpha}\ \sigma_{\dot\alpha}'(0)$$

$$\sigma_\alpha(0)\ \ -c(\mu)\ \tilde{\psi}^\mu(\bar{z}) \sim \bar{z}^{-\frac{1}{2}}\ \Gamma^\mu_{\alpha\dot\alpha}\ \sigma_{\dot\alpha}'(0)$$

where $c(\mu)$ is a constant depending on the index μ. For the eightbrane $c(\mu) = +1$ for $m \neq 9$ and -1 for $\mu = 9$ in NS sectors. This choice enforces the boundary condition $\psi^\mu = c(\mu)\tilde{\psi}^\mu$ for z on the negative imaginary axis and $\psi^\mu = -c(\mu)\tilde{\psi}^\mu$ for z on the positive imaginary axis. The boundary limit $\lim_{\mathrm{Re}\ z\to 0}\ \Sigma_\alpha$ of the bulk spin field has the correct OPE with the ψ's and $\tilde{\psi}$'s, so we can take this as a definition:

$$\sigma_\alpha \equiv \lim_{\mathrm{Re}\ z\to 0}\ \Sigma_\alpha$$

We define the $\sigma'_{\dot\alpha}$ similarly, in terms of boundary limits of $\Sigma'_{\dot\alpha}$'s. The operator $\lim_{\mathrm{Im}\ z\to 0} \Gamma^9_{\alpha\dot\alpha}\tilde\Sigma_{\dot\alpha}$ has the same OPE's with the ψ's and $\tilde\psi$'s as does σ_α. This OPE is consistent with the condition that $\Sigma_\alpha = \omega\ \Gamma^9_{\alpha\dot\alpha}\tilde\Sigma_{\dot\alpha}$ for any value of ω.

It is not possible for ω to be uniform over all sectors. We know this because there are vertex operators in the theory with odd winding and a single $\tilde\psi$ fermion. When such a vertex operator is transported past a boundary insertion of a bulk spin field, there is a -1 in the presence of $\tilde\Sigma$ and a $+1$ in the presence of a Σ. At the same time, the value of $X^9/2\pi R_9$ also changes by an odd number at the point of insertion. So the relative sign between Σ and $\tilde\Sigma$ at the boundary exactly depends on the boundary value of $\exp\{iX_i/2R_9\}$:

$$\sigma_\alpha \equiv \lim_{\mathrm{Re}\ z\to 0}\ \Sigma_\alpha = \exp\{iX_i/2R_9\}\ \Gamma^9_{\alpha\dot\alpha}\tilde\Sigma_{\dot\alpha}$$

This relation implies that the spin fields $\Sigma, \tilde\Sigma$ have zero modes $\Sigma_0, \tilde\Sigma_0$ on the interval exactly when the winding number, *i.e.* $(X_9 \ @\ \mathrm{L} - X_9\ @\ \mathrm{R})/2\pi R_9$ is even. This explains why the spectrum looks supersymmetric in the even winding sectors but nonsupersymmetric and potentially tachyonic in the odd winding sectors.

Though the phase ω varies from sector to sector, in all sectors it is the case that the boundary limits of the operator $\tilde\Sigma_{\dot\alpha}$ can be traded in for a boundary limit of the $\Gamma^9_{\dot\alpha\alpha}\Sigma_\alpha$, with some coefficient. This establshes the consistency of the multiplicative conservation, between bulk and boundary, of the quantity $(-1)^{F_{L_W}}\ \cdot\ (-1)^{F_{L_S}}$, the product of left-moving worldsheet and spacetime fermion parities. Though $(-1)^{F_{L_W}}$ and $(-1)^{F_{L_S}}$ cannot be conserved separately in the presence of a boundary, the combination is preserved by our boundary conditions. This is precisely the same as in the case of ordinary Dp-branes in type IIA with even p, but we emphasize it here because the context is less familiar due to the more involved choice of sectors. Below, we organize products of bulk and boundary operators into a multiplication table, so that the conservation of $(-1)^{F_{L_W}}\ \cdot\ (-1)^{F_{L_S}}$ is easy to check. It is precisely this quantity which is equal to $(-1)^w$ in all closed string sectors, and it can therefore be transmitted to open strings via the winding number.

Having defined the boundary spin fields σ_α, we can write the vertex operators for open strings in the RNS formalism. For the D8, we have the vertex operators are as follows (ghost and X^P dependence are omitted):

- Sector **a**: NS_+ states with even winding $w \in 2\mathbb{Z}$. The matter GSO is NS_-. This sector contains the gauge field, whose vertex operator is $\lim_{\mathrm{Re}\ z\to 0}\psi^M = \lim_{\mathrm{Re}\ z\to 0}\tilde\psi^M$. The transverse scalar is also present, with vertex operator $\lim_{\mathrm{Re}\ z\to 0}\psi^9 = -\lim_{\mathrm{Re}\ z\to 0}\tilde\psi^9$.

Sector	w mod 2	b.c. and full GSO
a	0	NS_+
b	0	R_+
c	1	NS_-
d	1	R_-

Table 4: Open string sectors in the background of an eightbrane in the Wilson line for $(-1)^{F_{L_S}}$.

	a	**b**	**c**	**d**
a	**a**	**b**	**c**	**d**
b	**b**	**a**	**d**	**c**
c	**c**	**d**	**a**	**b**
d	**d**	**c**	**b**	**a**

Table 5: Multiplication rules for open string vertex operators in the background of a D8-brane in the theory of a Wilson line for $(-1)^{F_{L_S}}$.

- Sector **b**: R_+ states with even winding $w \in 2\mathbb{Z}$. One such state is the massless fermion on the D8-brane, whose vertex operator is σ_α.
- Sector **c**: NS_- states with odd winding $w \in 2\mathbb{Z}+1$. The matter GSO is NS_+. This sector contains the tachyon, whose vertex operator is $\lim_{\mathrm{Re}\ z\to 0} E_{(0,w)}$, with w odd.
- Sector **d**: R_- states with odd winding $w \in 2\mathbb{Z}+1$. This sector contains massive winding fermions $\sigma'_{\dot\alpha}\ \lim_{\mathrm{Re}\ z\to 0} E_{(0,w)}$ which become light only in the limit $R_9 \to \infty$.

We summarize the open string sectors in table (4). The selection rules of the boundary OPE are summarized by the multipication table (5). There is also a consistent bulk-boundary OPE, as demonstrated by the following multiplication rules for bulk operators on boundary operators in table (6).

2.4 Lift to M-theory

It is in principle algorithmic to lift any Wilson line background to M-theory. A Wilson line background is simply string theory on a circle x^9, a topologically nontrivial space

	a	b	c	d
A, D	a	b	c	d
B, C	b	a	d	c
E, H	d	c	b	a
F, G	c	d	a	b

Table 6: Multiplication rules for open string vertex operators in the background of a D8-brane in the theory of a Wilson line for $(-1)^{F_{L_S}}$.

patched together from locally trivial pieces, using as a transition operation a discrete symmetry g of type IIA string theory. To find the corresponding M-theory background, lift the discrete symmetry operation to a symmetry g' of 11 dimensional M-theory on a circle x^{11}, and construct a circle x^9 patched together with the symmetry operation g'.

In this case $g = (-1)^{F_{L_S}}$, which lifts to M-theory as a reflection of x^{11}. This means our Wilson line background lifts to M-theory on a Klein bottle – a circle x^{11} fibered over another circle x^9 which reverses orientation $x^{11} \to -x^{11}$ as it is transported around x^9.

The lift illustrates previously unnoticed quantum properties of M-theory. In particular, the self-duality of type IIA on the twisted circle lifts to a new duality of M-theory which cannot be derived from any previously known dualities. In particular, though M-theory is known to be self-dual on a T^3, it is not dual to itself when compactified on a circle, torus, Möbius strip or cylinder.

Let us now work out the duality in M-theory terms. The parameters of the original M-theory are

$$m_{pl} = g_s^{-1/3} \alpha'^{-1/2}$$

$$R_{11} = g_s \alpha'^{+1/2}$$

Our duality changes the parameters by $R_9 \to \alpha'/(2R_9)$ and $g_s \to \sqrt{\frac{\alpha'}{2}}\, \frac{g_s}{R_9}$. So the parameters of the dual M-theory compactification are

$$R_{11}' = \left(\frac{R_{11}}{2} \right)^{1/2} m_p^{-3/2} R_9^{-1}$$

$$R_9' = \frac{1}{2}(R_9 R_{11} m_p^3)^{-1}$$

$$m_p' = (2R_{11})^{1/6} R_9^{1/3} m_p^{3/2}$$

As a check, note that the inverse string tension $R_{11}m_p^3$ and the nine-dimensional inverse Newton constant $m_p^9 R_9 R_{11}$ are invariant under this transformation.

The self-dual point is the point $R_9 = \sqrt{\alpha'/2}$. In eleven-dimensional terms this is when $R_{11}R_9^2 = \frac{1}{2\, m_p^3}$. In terms of massive charged states, this is the point where a wrapped membrane has the same tension as a mode with half a unit of Kaluza-Klein momentum along the R_9 direction. At the self-dual radius there is an enhanced gauge symmetry $SU(2) \times U(1)$, as we saw in a previous section. In the eleven-dimensional picture, the W-bosons of the enhanced gauge symmetry are membranes wrapping the Klein bottle and carrying half a unit of p_9. The emergence of enhanced gauge symmetry from wrapped membranes in M-theory is not new altogether; a related effect plays an important role in type II-heterotic duality, for instance [7]. However this is a surprisingly explicit example involving a completely flat background, illustrating an ubiquitous theme of string dynamics in a particularly simple setting.

3 An orbifold by reflection, with a $(-1)^{F_{L_S}}$ action

The Wilson line for $(-1)^{F_{L_S}}$ is in some sense a direct ingredient in the nongeometric string backgrounds of [6]. In this section we shall examine another type IIA background which constitutes a second direct ingredient of the theories described in [6].

Our second ingredient is a $\mathbb{Z}_2$ orbifold of four coordinates x^{6-9}, in which the orbifold action has an extra phase of $(-1)^{F_{L_S}}$ relative to the usual orbifold action. We shall see that this extra phase changes drastically the nature of the orbifold, altering the massless content of the six-dimensional theory on the singularity as well as the boundary conditions for bulk fields at the origin. The nature of the supersymmetry of the 6D theory will be changed relative to the usual orbifold. While the theory on an ordinary $\mathbb{R}^4/\mathbb{Z}_2$ singularity has $(1,1)$ supersymmetry in type IIA string theory, the theory on our modified singularity will have $(0,2)$ supersymmetry instead. The properties of branes will also be affected; the so-called 'regular' branes, which are pointlike in the four transverse directions, have instabilities sufficiently near the origin. And the 'pinned' branes[5], localized at the singularity, will have an odd number, rather than even, of infinitely extended directions.

[5]In the standard orbifold these would be called 'fractional' branes. However in our case we will see that they do not carry any charge of the bulk zerobrane, so the terminology 'fractional' is inappropriate here.

3.1 Closed string states

Now we will work out the closed string spectrum of the $R_{6789} \cdot (-1)^{F_{L_S}}$ orbifold. In the RNS formalism $(-1)^{F_{L_S}}$ acts only on spin fields for worldsheet fermions and not on ordinary worldsheet fields themselves. In the GS formalism, of course, the phase $(-1)^{F_{L_S}}$ acts on the $\tilde{\theta}$ variables. We could retrace the steps we took in the case of the Wilson line for $(-1)^{F_{L_S}}$ and work out the spectrum in the GS formlaism in light-cone gauge, later translating it to the RNS formalism. But having seen how this works in principle, we will not encumber the reader with an elaborate working of the same steps in this case. Instead we will proceed directly to the RNS description.

Untwisted sector

In the untwisted sector, the spectrum is simple to work out. The worldsheet coordinates X^i with $i = 6, 7, 8, 9$ have zero modes x^i and the physical states on the circle have a zero mode wavefunction factor $f(x^i)$ which is an eigenfunction under the Laplacian $-\sum_i \frac{\partial}{\partial x^{i2}}$. This Laplacian commutes with the operator R_{6789} which reflects X^i, so the physical states are common eigenstates of R_{6789} and k_i^2. They are also separately eigenstates of $(-1)^{F_{L_S}}$, and the orbifold projection in the untwisted sector simply demands that all states must have the same eigenvalues under both operators. Since one contribution to the eigenvalue of R_{6789} is determined by the change in sign of a wavefunciton upon reflection across the origin, the orbifold projection is most simply expressed in terms of boundary conditions on bulk fields.

The effect of $R_{6789} \cdot (-1)^{F_{L_S}}$ is to give a phase $\pm$ with the following contributions:

- A $-$ sign for wavefunctions which are odd upon reflection and a $+$ sign for wavefunctions which are even upon reflection. This is equivalent to a $-$ sign for modes which have 'D'-type boundary conditions at the origin, meaning that they vanish at $x^i = 0$, and a $+$ sign for modes which have 'N'-type boundary conditions, meaning that their partial derivatives $\partial_i f(x)$ vanish at $x^i = 0$.
- A $-$ sign for each Lorentz index oriented along the x^i directions.
- On spacetime fermions, a sign equal to the eigenvalue of Γ^{6789}. In type IIA this is equal to the eigenvalue Γ^{012345} for right-moving fermions and $-\Gamma^{012345}$ for left-handed fermions $\tilde{\Psi}$.
- From the $(-1)^{F_{L_S}}$ piece, a sign of (-1) for left-handed fermions and RR fields.

Totether, the last two contributions combine to give a sign on all fermions equal to the eigenvalue of Γ^{012345}, in the case of type IIA.

The result is that the fields

$$G_{MN}, B_{MN}, G_{ij}, B_{ij}, \Phi, C_i, C_{iMN}, C_{ijk}, P_+^{(6)}\Psi^M, P_+^{(6)}\tilde{\Psi}^M, P_-^{(6)}\Psi^i, \quad \text{and} \quad P_-^{(6)}\tilde{\Psi}^i$$

have N-type boundary conditions, while the fields

$$G_{Mi}, B_{Mi}, C_M, C_{MNP}, C_{ijM}, P_-^{(6)}\Psi^M, P_-^{(6)}\tilde{\Psi}^M, P_+^{(6)}\Psi^i, \quad \text{and} \quad P_+^{(6)}\tilde{\Psi}^i$$

have D-type boundary conditions. (Here $M, N, \cdots$ run from 0 to 5.)

Interpreted in terms of a choice of untwisted sectors on the worldsheet, this set of boundary conditions means that we have the usual orbifold projection in the untwisted NS_+/NS_+ and NS_+/R_+ sectors, and the *opposite* of the usual orbifold projection in the R_-/NS_+ and R_-/R_+ sectors. That is, in the R_-/NS_+ and R_-/R_+ sectors we keep states which are odd, rather than even, under R_{6789}.

Bulk fields with N-type boundary conditions have zero modes in six dimensions which are normalizable to the volume of the transverse space. Though these do not quite correspond to fluctuating six-dimensional modes, this is an artifact in some sense of the noncompactness of the local model. The zero modes of bulk fields with N-type boundary conditions are *almost* 6D fields, since one would expect them to become dynamical in 6D once the transverse space is compactified. Fields with D-type boundary conditions do not have zero modes at all in six dimensions, even normalized to the volume of the transverse space.

So in this sense, there are two six-dimensional gravitini with the *same* 6D chirality, corresponding to (0,2) SUSY in the 6D effective theory. This is in contrast to the usual 6D effective theory in the $\mathbb{R}^4/\mathbb{Z}_2$ orbifold in type IIA, which has (1,1) SUSY in six dimensions. Given this, we would expect that the normalizable modes in the 6D theory would be tensor, rather than vector, multiplets. In the next section this expectation will be borne out.

Twisted sector

In the twisted sector the fields X^i are antiperiodic on the string worldsheet, and the zero modes X_0^i do not exist. Their superpartners $\tilde{\psi}^i, \psi^i$ have the periodicity opposite what they would ordinarily have in the untwisted NS and R sectors.

Modular invariance under $\tau \to -\frac{1}{\tau}$ demands that we have exactly one twisted sector of each type: NS/NS, NS/R, R/NS, R/R, with the orbifold and chiral GSO projections determined by the requirement of level matching in every sector.

First let us determine the correct orbifold projections. In the twisted R/R sector, all worldsheet fields $X^i, \tilde{\psi}^i, \psi^i$ which are odd under R_{6789} are antiperiodic, and $L_0 - \tilde{L}_0$

vanishes in the oscillator ground state. Therefore the orbifold projection in the R/R sector must be $+$, for the $-$ states all have $L_0 - \tilde{L}_0 \in \mathbb{Z} + \frac{1}{2}$.

Taking the product of an even, twisted R/R state and the untwisted states and imposing closure of the algebra of vertex operators, we find that the twisted NS/NS and NS/R states must be – under the orbifold projection; and twisted R/NS states must be $+$.

The possible GSO projections in the twisted sector which are consistent with closure of the vertex algebra can be presented as $\mathrm{NS}_\sigma/\mathrm{NS}_{\sigma'}$, $\mathrm{NS}_\sigma/\mathrm{R}_{\sigma'}$, $\mathrm{R}_{-\sigma}/\mathrm{NS}_{\sigma'}$, $\mathrm{R}_{-\sigma'}/\mathrm{R}_{\sigma'}$, for some values of $\sigma\sigma' \in \{\pm 1\}$. The consistent values of σ, σ' can be derived from the requirements of level matching. Since the NS/NS sector is odd under R_{6789}, we must act on the vacuum with an odd number of the $\{\psi^i, \tilde{\psi}^i\}$. This proves that $\sigma\sigma' = -1$, so the consistent GSO must be of the form $\mathrm{NS}_\sigma/\mathrm{NS}_{-\sigma}$, $\mathrm{NS}_\sigma/\mathrm{R}_{-\sigma}$, $\mathrm{R}_{-\sigma}/\mathrm{NS}_{-\sigma}$, $\mathrm{R}_{-\sigma}/\mathrm{R}_{-\sigma}$, for some value of $\sigma \in \{\pm 1\}$.

So consider the sector $\mathrm{R}_{-\sigma}/\mathrm{NS}_{-\sigma}$, which is $\mathrm{R}_{-\sigma}/\mathrm{NS}_{+\sigma}$ in the matter sector. Including the superghost contribution, the ground state level mismatch is zero mod 1. The orbifold projection is $+$, which means $\#_{X^i} + \#_{\psi^i} + \#_{\tilde{\psi}^i} \in 2\mathbb{Z}$. Level matching mod 1 means that $\#_{\tilde{\psi}^i} + \#_{\psi^M} + \#_{X^i} \in 2\mathbb{Z}$. Adding these two equations gives $\#_{\psi^i} + \#_{\psi^M} \in 2\mathbb{Z}$ which means the matter GSO projection on the right must be $+$, so $\sigma = +1$.

Now let us find the lowest-lying states in each twisted sector.

- Twisted $\mathrm{NS}_+/\mathrm{NS}_-$: The ground state weight in the matter sector is $(1/4, 1/4)$ from the X^i and $(1/4, 1/4)$ from the $\psi^i, \tilde{\psi}^i$. So the total weight is $(1/2, 1/2)$, the correct weight for a massless physical state in the NS/NS sector. All the $\tilde{\psi}^M, \psi^M$ oscillators have nonzero energy, so the massless states must be scalars in 6 dimensions. The fermion zero modes $\tilde{\psi}^i_0, \psi^i_0$ generate a Clifford algebra represented by the ground states. Under the $SO(4) \simeq SU(2) \times SU(2)$ group of rotations of the X^i, the left-moving fermion ground states transform as $(2, 1)$ and the right-moving fermion ground states transform as $(1, 2)$. So the total set of ground states transforms as $(2, 2)$ under $SU(2) \times SU(2)$, which is the **4** of SO(4). We denote the massless twisted scalars by $\mathcal{M}^i$. Immediately it is clear that the $\mathcal{M}^i$ cannot be interpreted as geometric resolution moduli and B-field moduli through a collapsed two-cycle; geometric resolution and B-field moduli would transform in the $(1, 3)$ and $(1, 1)$ respectively under the $SU(2) \times SU(2)$ transverse rotations. The physical interpretation is interesting and puzzling. Does conformal perturbation theory permit the $\mathcal{M}^i$ to be given a finite expectation value? Given that they are massless and that the spacetime theory has sixteen supercharges, we expect that the answer must be affirmative. What is the physical interpretation of the theory with a finite value for $\mathcal{M}$? We cannot answer that directly at this

point. Later we shall consider type IIB on this background, S-duality will give us some insight into the nature of the resolved space. We will see that there is nonzero H-flux turned on when the resolution parameter is nonzero. As noted earlier, the orbifold projection is -, but this imposes no additional constraints beyond level matching.

- Twisted $\mathrm{NS}_+/\mathrm{R}_-$: the weight of the oscillator ground state is $(1/2, 5/8)$ in the matter sector, so the lowest states are massless. The $\tilde{\psi}^i$ are periodic with GSO projection $+$, and the ψ^i are antiperiodic, so the SO(4) representation of the spacetime fermions is (2,1). The ψ^M are periodic with GSO projection $-$, so the spacetime fermions transform as a SO(5,1) Weyl spinor with chirality $-$. Then level matched states satisfying the GSO projection all automatically survive the orbifold projection, which is opposite the usual one: they live in the subspace which has eigenvalue $-$ under R_{6789}. We denote the massless states by $\lambda^{\dot{A}}_{\dot{p}}$. Since the fermion zero modes are real, a reality condition can be imposed on this sector; the only such covariant condition is that the spinor be pseudo-Majorana $\bar{\lambda}^{\dot{A}}_p = C^*_{p\dot{p}} \epsilon^{\dot{A}\dot{B}} \lambda^{\dot{B}}_{\dot{p}}$.

- Twisted $\mathrm{R}_-/\mathrm{NS}_-$: the weight of the oscillator ground state is $(5/8, 1/2)$ in the matter sector, so the lowest states are massless. The ψ^i are periodic with GSO projection –, and the $\tilde{\psi}^i$ are antiperiodic, so the SO(4) representation of the spacetime fermions is (1,2). The $\tilde{\psi}^M$ are periodic with GSO projection $-$, so the spacetime fermions transform as a SO(5,1) Weyl spinor with chirality $-$. The orbifold projection is onto even states, and it is satisfied automatically for level matched states which satisfy the GSO projection. The spinor $\lambda^A_{\dot{p}}$ is pseudo-Majorana $\bar{\lambda}^A_p = C^*_{p\dot{p}} \epsilon^{AB} \lambda^B_{\dot{p}}$.

- Twisted $\mathrm{R}_-/\mathrm{R}_-$: The ground state weight of the matter theory is $(5/8, 5/8)$, so the lowest states are massless. The $\tilde{\psi}^i, \psi^i$ are antiperiodic, which means the $SO(4)$ representation of these states is trivial. The $\tilde{\psi}^M$ and ψ^M are periodic, so the RR fields transform in the chiral bispinor of SO(5,1) with both spinors having the same chirality. There are then four physical states at each momentum, corresponding to a RR scalar A and a two-form A_{MN} with self-dual field strength in six dimensions $dA_{MNP} = \frac{1}{6}\epsilon_{MNP}{}^{QRS} dA_{QRS}$. The orbifold projection onto even states is automatic in this sector, given level matching and the GSO projection.

Altogether the massless bosonic field content is a two-form with self-dual field strength, and five real scalars – exactly the bosonic content of a $(0, 2)$ tensor multiplet in six dimensions. Note that the $SO(4) \simeq SU(2) \times SU(2)$ is an R-symmetry of the (0,2) SUSY in 6D. One SU(2) factor is the obvious SU(2) rotating the two Weyl spinors,

and the second SU(2) is the quaternionic SU(2) performing phase rotations on a Weyl spinor in 6D, and rotating it into its conjugate. Once again we summarize the sectors in a table:

Sector	eigenvalue of R_{6789}	b.c. for X^i	b.c. for $\tilde{G}/G$ and full GSO	massless content
AA	$+$	untwisted	$\mathrm{NS}_+/\mathrm{NS}_+$	$G_{\mu\nu}, B_{\mu\nu}, \Phi$
BB	$+$	untwisted	$\mathrm{NS}_+/\mathrm{R}_+$	Ψ^{μ}_{α}
CC	$-$	untwisted	$\mathrm{R}_-/\mathrm{NS}_+$	$\tilde{\Psi}^{\mu}_{\dot{A}}$
DD	$-$	untwisted	$\mathrm{R}_-/\mathrm{R}_+$	$C_{\mu}, C_{\mu\nu\sigma}$
EE	$-$	twisted	$\mathrm{NS}_+/\mathrm{NS}_-$	$\mathcal{M}^i$
FF	$-$	twisted	$\mathrm{NS}_+/\mathrm{R}_-$	$\lambda^{\dot{A}}_{\dot{p}}$
GG	$+$	twisted	$\mathrm{R}_-/\mathrm{NS}_-$	$\lambda^{A}_{\dot{p}}$
HH	$+$	twisted	$\mathrm{R}_-/\mathrm{R}_-$	A_{MN}

3.2 Vertex operators

The construction of the vertex operators is straightforward. We present the forms of the massless vertex operators in the table below, using the following notation:

- τ is a ground-state twist operator for X^i
- $\Sigma_A, \Sigma'_{\dot{A}}$ are spin fields of each $SO(4)$ chirality for the fermions ψ^i. $\tilde{\Sigma}'_A$ and $\tilde{\Sigma}_{\dot{A}}$ are spin fields for the fermions $\tilde{\psi}^i$.
- $\Sigma_p, \Sigma'_{\dot{p}}$ are spin fields of each $SO(5,1)$ chirality for the fermions ψ^M. $\tilde{\Sigma}_p, \tilde{\Sigma}'_{\dot{p}}$ are spin fields of each $SO(5,1)$ chirality for the fermions $\tilde{\psi}^M$.

Every vertex operator has an additional factor of $\tilde{c}\, c\, \exp\{-\tilde{a}\tilde{\phi} - a\phi\}\, \exp\{ik_P X^P\}$, where $a, \tilde{a}$ are equal to 1 for NS sectors and $\frac{1}{2}$ for R sectors.

Checking the closure of the algebra of vertex operators is straightforward, keeping in mind:

- Two twist operators τ close on untwisted operators involving the X^i.
- Two Σ_A or two $\Sigma'_{\dot{A}}$ close on untwisted operators involving the ψ^i, of even fermion number, since they are spin fields for an even number of complex fermions. Likewise, two $\tilde{\Sigma}'_A$ or two $\tilde{\Sigma}_{\dot{A}}$ close on operators involving the $\tilde{\psi}^i$, of even fermion number.

field	matter vertex op.	sector	at $X^i = 0$
G_{MN}, Φ, B_{MN}	$f_+(X^i)\ \tilde{\psi}_M \psi_N$	$\mathrm{NS}_+/\mathrm{NS}_+\ ^{(+)}$	N
G_{ij}, B_{ij}	$f_+(X^i)\ \tilde{\psi}_i \psi_j$	$\mathrm{NS}_+/\mathrm{NS}_+\ ^{(+)}$	N
$G_{iI} + B_{iI}$	$f_-(X^i)\ \tilde{\psi}_i \psi_I$	$\mathrm{NS}_+/\mathrm{NS}_+\ ^{(+)}$	D
$G_{iI} - B_{iI}$	$f_-(X^i)\ \tilde{\psi}_I \psi_i$	$\mathrm{NS}_+/\mathrm{NS}_+\ ^{(+)}$	D
$P_6^{(+)} \Psi^M$	$f_+(X^i)\ \tilde{\psi}^M \Sigma_p \Sigma_A$	$\mathrm{NS}_+/\mathrm{R}_+\ ^{(+)}$	N
$P_6^{(-)} \Psi^M$	$f_-(X^i)\ \tilde{\psi}^M \Sigma'_{\dot{p}} \Sigma'_{\dot{A}}$	$\mathrm{NS}_+/\mathrm{R}_+\ ^{(+)}$	D
$P_6^{(-)} \Psi^i$	$f_+(X^i)\ \tilde{\psi}^i \Sigma'_{\dot{p}} \Sigma'_{\dot{A}}$	$\mathrm{NS}_+/\mathrm{R}_+\ ^{(+)}$	N
$P_6^{(+)} \Psi^i$	$f_-(X^i)\ \tilde{\psi}^i \Sigma_p \Sigma_A$	$\mathrm{NS}_+/\mathrm{R}_+\ ^{(+)}$	D
$P_6^{(+)} \tilde{\Psi}^M$	$f_+(X^i)\ \psi^M \tilde{\Sigma}'_A \tilde{\Sigma}'_p$	$\mathrm{R}_-/\mathrm{NS}_+\ ^{(-)}$	N
$P_6^{(-)} \tilde{\Psi}^M$	$f_-(X^i)\ \psi^M \tilde{\Sigma}_{\dot{A}} \tilde{\Sigma}_{\dot{p}}$	$\mathrm{R}_-/\mathrm{NS}_+\ ^{(-)}$	D
$P_6^{(-)} \tilde{\Psi}^i$	$f_+(X^i)\ \psi^i \tilde{\Sigma}_{\dot{A}} \tilde{\Sigma}_{\dot{p}}$	$\mathrm{R}_-/\mathrm{NS}_+\ ^{(-)}$	N
$P_6^{(+)} \tilde{\Psi}^i$	$f_-(X^i)\ \psi^i \tilde{\Sigma}'_A \tilde{\Sigma}'_p$	$\mathrm{R}_-/\mathrm{NS}_+\ ^{(-)}$	D
dC_{MN}	$f_-(X^i)\ \bar{\tilde{\Sigma}} \Gamma_{MN} \Sigma$	$\mathrm{R}_-/\mathrm{R}_+\ ^{(-)}$	D
dC_{ij}	$f_-(X^i)\ \bar{\tilde{\Sigma}} \Gamma_{ij} \Sigma$	$\mathrm{R}_-/\mathrm{R}_+\ ^{(-)}$	D
dC_{iM}	$f_+(X^i)\ \bar{\tilde{\Sigma}} \Gamma_{iM} \Sigma$	$\mathrm{R}_-/\mathrm{R}_+\ ^{(-)}$	N
dC_{MNPQ}	$f_-(X^i)\ \bar{\tilde{\Sigma}} \Gamma_{MNPQ} \Sigma$	$\mathrm{R}_-/\mathrm{R}_+\ ^{(-)}$	D
dC_{ijMN}	$f_-(X^i)\ \bar{\tilde{\Sigma}} \Gamma_{ijMN} \Sigma$	$\mathrm{R}_-/\mathrm{R}_+\ ^{(-)}$	D
dC_{ijkl}	$f_-(X^i)\ \bar{\tilde{\Sigma}} \Gamma_{ijkl} \Sigma$	$\mathrm{R}_-/\mathrm{R}_+\ ^{(-)}$	D
dC_{iMNP}	$f_+(X^i)\ \bar{\tilde{\Sigma}} \Gamma_{iMNP} \Sigma$	$\mathrm{R}_-/\mathrm{R}_+\ ^{(-)}$	N
dC_{ijkM}	$f_+(X^i)\ \bar{\tilde{\Sigma}} \Gamma_{ijkM} \Sigma$	$\mathrm{R}_-/\mathrm{R}_+\ ^{(-)}$	N
$\mathcal{M}^i$	$\tau\ \tilde{\Sigma}'^{\dagger}_A \Gamma^i_{A\dot{A}} \Sigma'_{\dot{A}}$	$\mathrm{NS}_+/\mathrm{NS}_-\ ^{(-)}$	localized
$\lambda^{\dot{A}}_{\dot{p}}$	$\tau\ \tilde{\Sigma}'_A \Sigma'_{\dot{p}}$	$\mathrm{NS}_+/\mathrm{R}_-\ ^{(-)}$	localized
$\lambda^{A}_{\dot{p}}$	$\tau\ \ \tilde{\Sigma}'_{\dot{p}}\ \Sigma_A$	$\mathrm{R}_-/\mathrm{NS}_-\ ^{(+)}$	localized
dA_{MNP}	$\tau\ \tilde{\Sigma}^{\dagger}{}_{\dot{p}} (\Gamma_{MNP})_{\dot{p}\dot{q}} \Sigma'_{\dot{q}}$	$\mathrm{R}_-/\mathrm{R}_-\ ^{(+)}$	localized

Table 7: Vertex operators for closed strings in the orbifold by R_{6789} with an action of $(-1)^{F_{L_S}}$.

	AA	BB	CC	DD	EE	FF	GG	HH
AA	**AA**	**BB**	**CC**	**DD**	**EE**	**FF**	**GG**	**HH**
BB	**BB**	**AA**	**DD**	**CC**	**FF**	**EE**	**HH**	**GG**
CC	**CC**	**DD**	**AA**	**BB**	**GG**	**HH**	**EE**	**FF**
DD	**DD**	**CC**	**BB**	**AA**	**HH**	**GG**	**FF**	**EE**
EE	**EE**	**FF**	**GG**	**HH**	**AA**	**BB**	**CC**	**DD**
FF	**FF**	**EE**	**HH**	**GG**	**BB**	**AA**	**DD**	**CC**
GG	**GG**	**HH**	**EE**	**FF**	**CC**	**DD**	**AA**	**BB**
HH	**HH**	**GG**	**FF**	**EE**	**DD**	**CC**	**BB**	**AA**

Table 8: Closure of the OPE in type IIA on $\mathbb{R}^{5,1} \times \mathbb{R}^4/\mathbb{Z}_2$ with an action of $(-1)^{F_{L_S}}$.

- A Σ_A and a $\Sigma'_{\dot{A}}$ close on an untwisted operator involving the ψ, with odd fermion number. Likewise a $\tilde{\Sigma}'_A$ and a $\tilde{\Sigma}_{\dot{A}}$ close on an untwisted operator involving the $\tilde{\psi}$, with odd fermion number.

- Two Σ_p or two $\Sigma'_{\dot{p}}$ close on untwisted operators involving the ψ^i, of odd fermion number, since they are spin fields for an odd number of complex fermions. Likewise, two $\tilde{\Sigma}_p$ or two $\tilde{\Sigma}'_{\dot{p}}$ close on operators involving the $\tilde{\psi}^i$, of odd fermion number.

- A Σ_p and a $\Sigma'_{\dot{p}}$ close on an untwisted operator involving the ψ, with even fermion number. Likewise a $\tilde{\Sigma}_p$ and a $\tilde{\Sigma}'_{\dot{p}}$ close on an untwisted operator involving the $\tilde{\psi}$, with even fermion number.

One can check these multiplication rules, if necessary, by grouping the real fermions arbitrarily in pairs into complex fermions, and performing a bosonization.

3.3 Type IIB version and S-duality

The type IIB version of this background is quite similar at weak coupling to the type IIA version. We shall describe it briefly, with some comments at the end about its S-dual version[6].

Bulk and localized spectrum

[6]See also [15], [16] for earlier work on the type IIB theory on the $(-1)^{F_{L_S}} \cdot R_{6789}$ orbifold, its S-dual, and its stable, non-BPS branes.

Type IIB string theory on the R_{6789} $(-1)^{F_{L_S}}$ orbifold can be obtained by starting with the type IIA theory on the same orbifold background, and performing a T-duality along one of the spacelike x^M directions. The spectrum of the resulting theory is that of type IIB in the bulk, with the following boundary conditions at the singularity:

- N-type boundary conditions for

$$G_{MN}, B_{MN}, G_{ij}, B_{ij}, \Phi, C_{iM}, C_{iMNP}, C_{ijkM}, P_+^{(6)}\Psi^M, P_-^{(6)}\tilde{\Psi}^M, P_-^{(6)}\Psi^i, \quad \text{and} \quad P_+^{(6)}\tilde{\Psi}^i$$

- D-type boundary conditions for

$$G_{Mi}, B_{Mi}, C, C_{MN}, C_{ij}, C_{MNPQ}, C_{MNij}, C_{ijkl}, P_-^{(6)}\Psi^M, P_+^{(6)}\tilde{\Psi}^M, P_+^{(6)}\Psi^i, \quad \text{and} \quad P_-^{(6)}\tilde{\Psi}^i$$

The boundary conditions for the gravitini indicate that the global SUSY of the 6D theory on the singularity is of type $(1,1)$. The localized states do indeed organize themselves with respect to this supersymmetry; the twisted sector contains a RR vector, and one Weyl fermion of each chirality. This is the content of a vector multiplet of $(1,1)$ supersymmetry in 5+1 dimensions.

S-duality to the O5$^-$–D5 system

In the type IIB case, we can apply S-duality to obtain a more familiar description. Conjugating the $(-1)^{F_{L_S}}$ by S-duality, we find that the boundary condition for bulk fields in the new description is that they are periodic or antiperiodic according to their transformation properties under $R_{6789} \cdot \Omega$, where Ω is the worldsheet parity transformation which acts with a minus sign on $B_{\mu\nu}, C, C_{\mu\nu\sigma\tau}$, and $\Psi - \tilde{\Psi}$ leaving $C_{\mu\nu}$ and all other massless NS-NS fields invariant, including $\Psi + \tilde{\Psi}$.

The reader will recognize this boundary condition as the boundary condition at an orientifold 5-plane of the $\widetilde{\text{IIB}}$ theory. However the O5 plane cannot be the only object present. The amount of spactime SUSY of the oriented IIB theory on the $(-1)^{F_{L_S}}$ orbifold forbids any corrections to the energy of the background, even nonperturbatively. The O5 plane has a nonzero tension, positive or negative depending whether it is an O5$^+$ or an O5$^-$, while the S-dual of our orbifold fixed plane has zero tension. This suggests that the dual $\widetilde{\text{IIB}}$ theory must have another object to cancel the tension of the O5 plane, namely a single D5, with the sign of the O5 plane constrained to be O5$^-$. Adding a D5 to the O5$^-$ in the $\widetilde{\text{IIB}}$ background also gives a moduli space which for small separations matches that of the infinitestimal twisted deformations of the IIB theory on the $(-1)^{F_{L_S}}$ orbifold. That is to say, the moduli space is four dimensional, and the deformations transform in the **4** of the transverse rotation group. In the $\widetilde{\text{IIB}}$ theory these deformations are just the motions of the D5-brane away from the O5$^-$.

For purposes of the next section it will be interesting to take note of the spectrum of open strings stretching from the D5 to itself, at a point in moduli space where the fivebrane is pulled away from the origin by a distance $r << \sqrt{\alpha'}$. Since there is only a single D5, the lowest open string modes stretching from the D5, through the O5$^-$ plane to the image D5, are projected out by the Ω transformation. That is to say, the modes which are projected out are the ones with masses of order r^1 for small r, which would become massless with the fivebrane at the origin. Let us review the reason for that.

As explained in [4], the orientifold projection of an Op$^-$-plane on the oscillator ground state of the Dp-Dp open string sectors yields states which are antisymmetric in Chan-Paton indices. For k D5-branes the number of stretched open strings in the ground state which pass through the O-plane is $k(k-1)$, and they live in the antisymmetric tensor representation of $U(k)$. These ground state open strings are BPS and live in massive vector multiplets; when the D5-brane is coincident with the O5$^-$-plane they enhance the gauge symmetry from $U(k)$ to $SO(2k)$. But for $k = 1$ the number of such light open strings is zero, and going from $U(1)$ to $SO(2)$ does not enlarge the dimension of the continuous group.

The absence of BPS open strings stretching from the D5 to itself through the O5$^-$ plane supports the S-duality between the IIB and $\widetilde{\text{IIB}}$ backgrounds. The $U(1)$ propagating on the D5-brane of the $\widetilde{\text{IIB}}$ theory becomes the $U(1)$ of the twisted Ramond-Ramond vector field of the IIB side. If the gauge group were enhanced to something nonabelian at the origin, the continuous gauge symmetries on the two sides would not agree.

However there are still *non*-BPS open strings stretching from the D5 to itself through the O5$^-$. An open string state can survive the Ω-projection if the oscillators contribute a $-$ sign in place of the Chan-Paton indices. Such modes live in the rank-2 symmetric tensor representation of the $U(k)$ group, and for a single D5 there is one of them for each oscillator state with Ω eigenvalue -1. Though these strings lack supersymmetry, the lightest is always absolutely stable. All the stretched open string states have the same, nonzero charge ± 1 under the $U(1)$ gauge symmetry.

For small separations from the origin, the charged stretched states do not become massless; they retain masses $m^2 = o(\frac{1}{\alpha'})$ due to their oscillator energies. For large separations of the fivebrane from the origin $r >> \sqrt{\alpha'}$ all open string states, in particular the lightest, have masses which scale like $m^2 \sim \frac{r^2}{4\pi^2\alpha'}$. This formula is true at tree level, and the locally BPS nature of the stretched strings guarantee that the leading piece of the mass is uncorrected at strong coupling.

The presence of charged, non-BPS fundamental strings fixed to the D5-brane in the $\widetilde{\text{IIB}}$ side indicates that there should be charged, non-BPS D-branes pinned to the fixed

locus in the IIB orbifold, the lightest one being absolutely stable. The presence of a massless RR field suggests as much; in the next section we will construct the stable non-BPS branes explicitly.

3.4 Branes and open strings in the $R_{6789} \times (-1)^{F_{L_S}}$ orbifold

General comments

The $R_{6789} \times (-1)^{F_{L_S}}$ orbifold will prove to be quite similar to the Wilson line in one important respect: the ordinary branes ('regular' branes in the language of [8]), localized in the transverse space away from the fixed point, cannot be absolutely stable. This can be seen at many levels. First of all, the timelike component C_0 of the RR vector has D-type boundary conditions at the fixed point, meaning that it has no zero mode in six dimensions, even one normalized to the volume of the transverse space. Since the zero mode of a gauge field is the Lagrange multiplier which enforces charge conservation, its absence suggests that zerobrane charge need not be conserved. Second of all, parallel transporting a brane around the fixed point brings it back as its own antibrane, which violates integer-valued charge conservation. Finally we shall see that the zerobrane becomes unstable perturbatively when moved sufficiently close to the fixed point. Much of the discussion is parallel to the case of the Wilson line for $(-1)^{F_{L_S}}$ and we will move through it as tersely as possible.

On the other hand, we will see that in the sector of branes pinned to the origin – described in the context of the pure R_{6789} orbifold as 'fractional' branes – there are Dp-branes with odd p which are non-BPS but *stable* , unlike the usual wrong-dimension branes of type IIA string theory.

The regular brane

The regular brane is straightforward; we can construct it in the usual way [8], by taking a brane and its image on the covering space. The inclusion of the action $(-1)^{F_{L_S}}$ means that the configuration on the covering space is a brane, along with its antibrane at the reflected point in the X^i space. For definiteness, consider a D0-brane, though branes extended in some of the spacelike X^M directions can be obtained by taking T-duals along X^M.

On a single brane, the massless bosons are a gauge field A_0^{D0}, five transverse scalars $X^M_{D0}, M \neq 0$ and four more transverse scalars X^i_{D0}. There are also sixteen fermions which transform as a Majorana spinor of SO(9). On an antibrane the content is the same. Stretching between them is a complex scalar string T whose mass is

$$m_T^2 = \tfrac{1}{4\pi^2\alpha'^2} \left[\; (X^M_{D0} - X^M_{\bar{D}0})^2 + (X^i_{D0} - X^i_{\bar{D}0})^2 \; \right] - \tfrac{1}{\alpha'}$$

The effect of the orbifold projection is to set $X^M_{D0} = X^M_{\bar{D}0} \equiv X^M$ and $X^i_{D0} = -X^i_{\bar{D}0} \equiv X^i$, as well as $A_0^{D0} = A_0^{\bar{D}0}$. The tachyon T transforms to its conjugate T^* under $(-1)^{F_{L_S}}$, so the orbifold projection constrains the tachyon to be real, $T = T^*$. One linear combination of the massless fermions is also projected out by the orbifolding. This leaves a $U(1)$ gauge theory with nine neutral scalars X^M, X^i, a single massless SO(9) Majorana spinor of fermions, and a charged scalar with mass

$$m_T^2 = \tfrac{1}{\pi^2\alpha'^2}\left[\ X^{i2} - \pi^2\alpha'\ \right]$$

When the zerobrane comes within a distance $\pi\sqrt{\alpha'}$ of the origin, then, it becomes tachyonic and can decay. This is consistent with our observation that there is no zero mode of the Ramond-Ramond Coulomb potential C_0. It would be interesting to understand the end product of the decay, whether it is the closed string vacuum or whether some remnant, possibly carrying a discrete charge, may survive.

Stable non-BPS branes pinned to the fixed plane

Our theory contains a RR tensor localized to the fixed plane $X^i = 0$; this suggests that the theory should contain charged p-branes of odd p which are pinned to the fixed plane. Such branes should show up as 'fractional' branes in the sense of [8]. We would like to avoid that terminology in this example, however, since the brane which is pinned to the fixed locus does not carry charge under any bulk RR field.

We now choose to construct the open string Hilbert space of the pinned D5-brane, since it preserves the full Lorentz invariance of the orbifold background; the D3 and D1 branes can be obtained by T-dualities along the X^M directions. The boundary condition at the endpoints is $\partial_n X^M = X^i = 0$ for the bosons. The boundary condition for the supercurrents is $G = \tilde{G}$ at the left endpoint, with $G = \pm\,\tilde{G}$ at the right endpoint in the R and NS sectors, respectively. So the boundary condition on the fermions is $\psi^M = \tilde{\psi}^M, \psi^i = -\,\tilde{\psi}^i$ at the left endpoint, and $\psi^M = \pm\,\tilde{\psi}^M, \psi^i = \mp\,\tilde{\psi}^i$ at the right endpoint in the R and NS sectors, respectively.

Now we would like to understand which GSO and orbifold projections must be imposed. In particular, we expect that there should not be a tachyon, since the brane is charged and there is no obvious lighter state which carries the same charge.

Let us take a moment to recall why stable fivebranes are not allowed in the ordinary type IIA theory, fractional or otherwise. Starting with an open string vertex operator in the NS_+ sector, we could take its OPE with a RR vertex operator in the bulk. Since the bulk RR operators have GSO projection $\mathrm{R}_-/\mathrm{R}_+$ in type IIA, this would give an open string with NS boundary conditions, but GSO projection –. Thus one is compelled to include both values + and – of $(-1)^{F_W}$ in both the NS and R open string

sectors. In particular the open string NS$_-$ sector contains a real tachyon. Thus one is left with the theory of an unstable, neutral brane rather than a charged, stable brane.

For the case of the type IIA orbifold by R_{6789} $(-1)^{F_{L_S}}$, the bulk fields in the R$_+$/R$_-$ sector have orbifold projection $-$ rather than $+$. This means that if we start with an open string in the NS$_+$ sector and take its OPE with one of the untwisted R$_-$/R$_+$ states, we end up with an NS sector with GSO projection - and orbifold projection $-$. The matter GSO of such an open string is $+$, so the lowest states satisfying the $-$ orbifold projection are obtained by acting on the vacuum with X^i_{-1} or with $(\psi^i - \tilde{\psi}^i)_{-\frac{1}{2}}$ and $(\psi^M + \tilde{\psi}^M)_{-\frac{1}{2}}$. To get a matter state of weight $\frac{1}{2}$, one must add momentum k_M corresponding to a mass squared of $+\frac{1}{\alpha'}$. So there are no tachyons in the NS$_+$ sector.

More generally, total worldsheet fermion parity $(-1)^{F_W}$ is equal to R_{6789}-parity for *all* closed string states, twisted and untwisted. It follows that we get a consistent OPE between the bulk and the boundary if we apply the same correlated projection to all open string states as well. The resulting sectors are:

- NS$_+$, with orbifold projection $+$. The GSO projection in the matter sector is then $-$, so the lowest allowed states are $(\psi^M + \tilde{\psi}^M)_{-\frac{1}{2}}\,|0\rangle$, which give the states of a massless gauge field a_M living on the brane.

- NS$_-$, with orbifold projection $-$. The GSO projection in the matter sector is $+$, and the lowest states surviving the projection are massive, as discussed above.

- R$_+$, with orbifold projection $+$. The fermions all have zero modes in this sector, $(\psi^M + \tilde{\psi}^M)_0$ for the M fermions and $(\psi^i - \tilde{\psi}^i)_0$ for the i fermions. The projections mean that the product of the M zero modes is $+1$ and the product of the i zero modes is $+1$. So we get a spacetime fermion μ^A_p which transforms as $(1, 2)$ under SO(4) and as a Weyl fermion of positive chirality under $SO(5, 1)$.

- R$_-$, with orbifold projection $-$. The fermions all have zero modes in this sector. The projections mean that the product of the M zero modes is $+1$ and the product of the i zero modes is -1. So we get a spacetime fermion $\mu^{\dot{A}}_p$ which transforms as $(2, 1)$ under SO(4) and as a Weyl fermion of positive chirality under $SO(5, 1)$.

We had a choice of two GSO projections in the open string R sectors; our choice was dictated by the necessity that the OPE of the gravitino vertex operator for $P^{(+)}\Psi^M$ with the open string vertex operator in the NS sector be consistent. That is, R sector GSO is fixed by the reqirement that the open string R sector vertex operators contain only Σ_p's and not $\Sigma_{\dot{p}}$'s.

In addition to the stable D5, there is also a stable D1 and D3 in this theory which are pinned to the origin $X^i = 0$. The tree-level dynamics of the stable D3 and D1 can

be obtained by performing T-dualities along pairs of X^M directions.

Tadpoles, boundary states and SUSY breaking by the brane

Earlier we claimed in passing that the pinned branes are not charged under any bulk RR field. This is intuitively obvious, since there are no bulk RR fields with the correct Lorentz properties to couple to the pinned p-branes, but we can demonstrate the decoupling from bulk RR fields directly, in terms of boundary states.

The projection on the space of open string states is $(-1)^{F_W}\ R_{6789}$, a combination of worldsheet fermion number mod 2, and the geometric action of the reflection. Since it is a single projection and not two independent projections, the worldsheet partition function can be described by a sum over two sets of boundary conditions rather than four. That is, the partition function [2] at modular parameter t is

$$V_6 \int \tfrac{d^6k}{(2\pi)^6}\ \tfrac{1}{2}\ \mathrm{tr}_{\mathcal{H}^{\perp}_{\text{open}}} \Big[\ (-1)^{F_W}\ \Big(\ 1 + (-1)^{F_W} R_{6789}\ \Big)\ \exp\{-Ht\}\ \Big].$$

The first factor of $(-1)^{F_W}$ is the usual thermal boundary condition for worldsheet fermions (it can also be thought of as coming from the $(-1)^{F_W}$ contribution of the superghosts), and the $\frac{1+(-1)^{F_W} R_{6789}}{2}$ implements the correlated GSO/orbifold projection. The trace runs over all states of the oscillators transverse to k^M.

So the partition function breaks up into two sectors, one with antiperiodic boundary conditions for fermions and periodic boundary conditions for the X^μ, and the other sector with periodic boundary conditions for $X^M, \psi^M, \tilde{\psi}^M$ and antiperiodic boundary conditions for $X^i, \psi^i, \tilde{\psi}^i$. Re-interpreted in the closed string channel (as in [12],[13]), where the space and Euclidean time coordinates are reversed relative to the open string channel, the boundary state corresponding to the D-brane has two contributions. One is from the sector with X^i untwisted, with NS boundary conditions for the supercurrents. The other is from the sector with X^i antiperiodic and Ramond boundary conditions for both supercurrents $G, \tilde{G}$. The target space interpretation is that the pinned branes at the fixed locus are sources for the bulk NS/NS fields (such as $G_{\mu\nu}$) and the twisted RR fields (such as A_{MN}) but not the twisted NS/NS fields (such as $\mathcal{M}^i$) or the bulk RR fields (such as C_μ or $C_{\mu\nu\sigma}$).

The absence of a one-point function for twisted NS/NS states can be derived from symmetry as well. All twisted NS/NS states transform in odd-rank tensor representations of $SO(4)$, so in particular there is no singlet which could get a tadpole. When we give an expectation value to the $\mathcal{M}^i$, twisted and untwisted fields can mix in the closed string theory, so we should expect that the deformed D-brane state should source all possible NS/NS and R/R fields when $\langle \mathcal{M}^i \rangle \neq 0$. This agrees with our picture of the stable pinned zerobrane on the IIB side coming from the $\widetilde{\text{IIB}}$ S-dual; if the brane is dual to

a stretched excited string state, its mass should go like $m \sim \sqrt{(\text{const.}) \cdot \mathcal{M}^i \mathcal{M}^i + m_0^2}$. So the tadpole $\frac{\partial m}{\partial \mathcal{M}^i}$ is nonzero at a generic point in moduli space, but vanishes at $\mathcal{M}^i = 0$, which is where we can calculate the partition function.

We end the section with a brief comment on the supersymmetry properties of the stable branes. Unlike the case of the Wilson line for $(-1)^{F_{L_S}}$, the type IIA orbifold by $R_{6789} \cdot (-1)^{F_{L_S}}$ has massless RR gauge fields propagating in six dimensions, indicating that the odd dimensional p-branes in the singular locus are charged and can be stable. Indeed, there is no tachyon in the spectrum of these branes. Despite being stable, they are not BPS. The only bosonic massless field propagating on the branes is a $U(1)$ vector, but there are two sets of massless Weyl fermions in the spectrum rather than one. Thus it is clear that the stable pinned branes break all the supersymmetry of the orbifold background. In fact, our consistency condition for the choice of R sectors encoded the fact that the global SUSY of the background broken by the brane is $(0, 2)$, and the goldstone fermions should have the same chirality as the corresponding gravitini.[7]

Vertex operators

The vertex operators for open strings on branes work out straightforwardly. For the regular brane, the open string Hilbert space and vertex operators are just projections of those for ordinary branes and antibranes on the covering space. This makes the construction of vertiex operators elementary for the regular branes and we shall not review it.

For the stable pinned branes we now list the sectors in the physical Hilbert space:

- In sector **aa** we have NS_+ states (matter GSO NS_-). The orbifold projection is $+$. The massless gauge field a_M has vertex operator is $\psi_M = \tilde{\psi}_M$.
- In sector **bb** we have R_+ states (matter GSO $-$). The orbifold projection is $+$. The massless state $\mu_{\dot{p}}^{A}$ has vertex operator $\lim_{\mathrm{Re}\ z \to 0} \Sigma'_{\dot{p}} \Sigma_A$.
- In sector **cc** we have R_- states (matter GSO $-$). The orbifold projection is $-$. The vertex operator for the massless state $\mu_{\dot{p}}^{\dot{A}}$ is $\lim_{\mathrm{Re}\ z \to 0} \Sigma'_{\dot{p}} \Sigma_{\dot{A}}$.
- In sector **dd** we have NS_- states (matter GSO NS_+). The orbifold projection is $-$. There are no massless states in this sector.

The open string vertex operators have a consistent multiplication among themselves and with closed string vertex operators, as summarized by table (9).

[7] This in turn is opposite to the chirality of the corresponding supercharges.

	aa	bb	cc	dd
aa	**aa**	**bb**	**cc**	**dd**
bb	**bb**	**aa**	**dd**	**cc**
cc	**cc**	**dd**	**aa**	**bb**
dd	**dd**	**cc**	**bb**	**aa**
AA,**HH**	**aa**	**bb**	**cc**	**dd**
BB,**GG**	**bb**	**aa**	**dd**	**cc**
CC,**FF**	**cc**	**dd**	**aa**	**bb**
DD,**EE**	**dd**	**cc**	**bb**	**aa**

Table 9: Multiplication rules for open and closed string vertex operators in the background of the stable pinned brane at the fixed locus of the orbifold by R_{6789} with an action of $(-1)^{F_{L_S}}$.

Type IIB version

The branes of the type IIB theory on the $(-1)^{F_{L_S}}$ R_{6789} orbifold can be understood simply from T-dualizing the type IIA version, along an odd number of the spacelike x^M directions. The behavior of the IIB branes is, not surprisingly, parallel to that of the IIA branes. The regular branes are perturbatively stable far from the fixed point, developing an instability when they come within a distance $\pi\sqrt{\alpha'}$ of the origin. There are fractional zerobranes and twobranes which are electrically and magnetically charged under the twisted RR vector multiplet, as well as a fourbrane which is charged under the (nondynamical) twisted RR five-form. These branes are all non-BPS. Since they are the lightest objects charged under their respective gauge potentials, they must be absolutely stable, not just perturbatively.

The existence of the non-BPS pinned zerobrane in type IIB orbifold background is an interesting check on the duality to the O5$^-$+D5 system in the type $\widetilde{\mathrm{IIB}}$ dual. As we pointed out earlier the D5, when separated from the O5$^-$, has open strings stretching from itself, through the O5$^-$-plane to itself again. These strings never become massless, even when the D5 returns to the origin. They are charged under the gauge field on the D5 and the lightest is absolutely stable. S-duality then dictates the existence of a stable, non-BPS object charged under the twisted RR one-form. The stable non-BPS zerobrane of IIB on the $(-1)^{F_{L_S}}$ orbifold has exactly the right properties to be the dual state.

4 Discussion

In this paper we have presented two backgrounds of type II string theory which are exactly solvable at tree level and preserve 16 supercharges. Both backgrounds display new phenomena and illuminate unexpected aspects of highly supersymmetric theories of quantum gravity. We have focused on type IIA, but the properties of type IIB on the same backgrounds can always be studied by performing a T-duality along one of the trivial directions.

The first background, a Wilson line for $(-1)^{F_{L_S}}$, displays a self-duality – as opposed to a duality to the opposite type II theory – on a single circle, as well as enhanced gauge symmetry at the self-dual radius. In type IIA string theory, the operation $(-1)^{F_{L_S}}$ is simply a reflection of the M-circle (together with the operation $C_{\mu\nu\sigma} \to -C_{\mu\nu\sigma}$ on the M-theory three-form), so the Wilson line background can be interpreted as M theory on a Klein bottle. Given that interpretation, the self-duality and appearance of enhanced gauge symmetry are quite surprising and new.

We have been forced to conclude that M-theory on a Klein bottle has a self-duality which changes the volume of the Klein bottle and maps wrapped twobranes into Kaluza-Klein modes. The duality is remaniscent of the self-duality of M-theory on a T^3 [14], but logically distinct from it, nor can the former be derived from the latter. Since the duality maps ordinary, locally charged D-branes to unstable, uncharged branes, this connection might be used to shed light on the place of non-BPS branes in M-theory.

Our second background is type II string theory on $\mathbb{R}^4/\mathbb{Z}_2$, where the $\mathbb{Z}_2$ acts as $(-1)^{F_{L_S}}$ in addition to the geometric action R_{6789}. This background has the field content of type IIA/B in the bulk, but the localized modes and fractional branes are those of type IIB/A. In neither case are the 'regular' branes, pointlike and separated from the fixed point, absolutely stable, though sufficiently far from the fixed point they are tachyon-free at tree level.

These orbifolds have twisted marginal operators which can be integrated to finite marginal directions. The nature of the marginal deformation of the CFT is unknown, but is possibly nongeometric in the sense of [6]. Indeed, the orbifold arises in a certain decompactification limit of the $\hat{c} = 4$ example worked out in that paper.

In type IIB, the marginally deformed orbifold can be S-dualized to a D-fivebrane pulled away from an $O5^-$-plane. Therefore the deformed orbifold contains an NS-fivebrane and has nonvanishing H-flux. The direct interpretation in type IIA is far from clear, though the NS/NS content of the IIA version and the IIB version must be the same. The presence of H-flux in the deformed orbifold agrees with what one would generally expect from the T^2 fibration picture of [6]. In those examples, H-flux always appears when one deforms away from the asymmetric orbifold point. However

the decompactification does not commute straightforwardly with the deformation of the orbifold, since the volume of the T^2 fiber is not a free parameter in the compact deformed orbifold. Further study would help to make the connection with nongeometric string theories more completely clear.

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